المادة: بحوث عمليات Ch1

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Chapter one

An introduction to Linear programming

1-Definition of Linear Programming

Linear programming is a simple technique ,it used for problems associated with **optimization**. We optimize a scenario based upon a number of constraints which govern that scenario. In business, we can use it to maximize profit or minimize costs based upon the resources available to any company

2- Applications of linear programming

Applications of linear programming are every where around you. You use linear programming at personal and professional fronts. You are using linear programming when you are driving from home to work and want to take the shortest route. Or when you have a project delivery you make strategies to make your team work efficiently for on time delivery. In business, we can use it to maximize profit or minimize costs based upon the resources available to any company. It can be implemented on manufacturing, transportation of commodities, allocation of resources etc.

3-Importance of linear programming

There are many benefits of linear programming for business

1. **Solve the business problems**
   With linear programming we can easily solve business problem. It is very benefited for increase the profit or decrease the cost of business.

2. **Easy work of manager under limitations and condition :-**
   Linear programming solve problem under different limitations and conditions , so It is easy for manager to work under limitations and conditions . It helps manager to decide in different limitations.

3. **Use in solving staffing problems:-**

With linear programming , we can calculate the number of staff needed in hospitals ,mines , hotels and other type of business.

4. Helpful in profit planning :-
Today linear programming is using for good profit planning.

5. Select best advertising media
With linear programming we can select best advertising media among a numbers of media.

6. solve the diet problems :-
With linear programming you can solve the diet problems with minimum cost. It is very useful for hospitals .There are different elements like vitamins, proteins, carbohydrates etc. You can select best quantity of them with minimum cost.

4 –formulation of the linear programming model

The formula of the linear programming model component are:-

- **Decision Variables:** The decision variables are the variables which will decide my output. They represent my ultimate solution. To solve any problem, we first need to identify the decision variables..
- **Objective Function:** It is defined as the objective of making decisions.
- **Constraints:** The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables..
- **Non-negativity restriction:** For all linear programs, the decision variables should always take non-negative values. Which means the values for decision variables should be greater than or equal

3- TERMINOLOGY USED IN LINEAR PROGRAMMING PROBLEM

1. Components of LP Problem: Every LPP is composed of a. Decision Variable, b. Objective Function, c. Constraints.

2. Optimization: Linear Programming attempts to either maximize or minimize the values of the objective function.

3. Profit of Cost Coefficient: The coefficient of the variable in the objective function express the rate at which the value of the objective function increases or decreases by including in the solution one unit of each of the decision variable

4. Constraints: The maximization (or minimization) is performed subject to a set of constraints. Therefore LP can be defined as a constrained optimization problem. They reflect the limitations of the resources.

5. Input-Output coefficients: The coefficient of constraint variables are called the Input/output Coefficients. They indicate the rate at which a given resource is unitized or depleted. They appear on the left side of the constraints.

6. Capacities: The capacities or availability of the various resources are given on the right hand side of the constraints

The mathematical expiration the LP model

The general LP Model can be expressed in mathematical terms as shown below:

Let Oij = Input-Output Coefficient

Cj = Cost (Profit) Coefficient

bi = Capacities (Right Hand Side)

Xj = Decision Variables

Find a vector (x1, x2, x3 ....xn) that minimize or maximize

a linear objective function F(x)

where F(x) = c1x1 + c2x2 + ...+ cnxn

subject to linear constraints

a1x1 + a2x2 + ...+ anxn ≤ b2

a1x1 + a2x2 + ...+ anxn ≤ b2

....

....

am1x1 + am2x2 + ...+ amnxn ≤ b2

and non-negativity constraints

x1 ≥ 0, x2 ≥ 0, ..., xn ≥ 0

<u>EXAMPLE 1</u>(PRODUCTION ALLOCATION PROBLEM)

A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below:

Machine	Time per unit (Minutes)			Machine Capacity
	Product 1	Product 2	Product 3	(minutes/day)
M_1	2	3	2	440
M_2	4	-	1	470
M_3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs.= 4, Rs= .3 and Rs=.6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit. Formulation of Linear Programming Model

Solution :-

There will be threee constraints. One for machine minutes availability. Then Decision variables

X1 = Number of units of A manufactured per daily

X2 = Number of units of B manufactured per daily.

X_3 = Number of units of B manufactured per daily

The objective function :

$$\text{Max } Z = 4x1 + 3x2 + 6x3$$

Subjective Constraints:

$$2x1 + 3x2 + 2x3 \leq 440$$

$$4x1 + 0x2 + 3x3 \leq 470$$

$$2x1 + 5x2 + 0x3 \leq 430$$

Non negativity x1, x2 ,x3>0

EXAMPLE 2 :

A factory manufactures two products A and B. To manufacture one unit of A, 1.5 machine hours and 2.5 labour hours are required. To manufacture product B, 2.5 machine hours and 1.5 labour hours are required. In a month, 300 machine hours and 240 labour hours are available. Profit per unit for A is Rs. 50 and for B is Rs. 40. Formulate as LPP

Solution:

Products	Resource/unit	
	Machine	Labour
A	1.5	2.5
B	2.5	1.5
Availability	300 hrs	240 hrs

Solution:

There will be two constraints. One for machine hours availability and for labor hours availability. Decision variables

X1 = Number of units of A manufactured per month

X2 = Number of units of B manufactured per month.

The objective function :

$$\text{Max } Z = 50\, x1 + 40\, x2$$

Subjective Constraints:

For machine hours

$$1.5x1 + 2.5x2 \le 300$$

For labor hours

$$2.5x1 + 1.5x2 \le 240$$

Non negativity $x1, x2 \ge 0$

EXAMPLE: 3

A company produces three products A, B, C. For manufacturing three raw materials P, Q and R are used. Profit per unit

A - Rs. 5, B - Rs. 3, C - Rs. 4

Resource requirements/unit

Raw Material / Product	P	Q	R
A	-	20	50
B	20	30	-
C	30	20	40

Maximum raw material availability:

P - 80 units, Q - 100 units; R - 150 units. Formulate LPP.

Decision variables:

x1 = Number of units of A

x2 = Number of units of B

x3 = Number of units of C

Objective Function

Since Profit per unit is given, objective function is maximization

Max $Z = 5x1 + 3x2 + 4x3$

Constraints:

For P: $0x1 + 20x2 + 30x3 \le 80$

For Q: $20x1 + 30x2 + 20x3 \le 100$

For R: $50x1 + 0x2 + 40x3 \le 150$

(for P, Q, R is not required) $X1, X2, X3 \ge 0$

Chapter two

Linear programming solution

1- Graphical methods

Introduction:-

There are two methods available to find optimal solution to a Linear Programming Problem. One is graphical method and the other is simplex method. Graphical method can be used only for a two variables problem i.e. a problem which involves two decision variables. The two axes of the graph (X & Y axis) represent the two decision variables X1 & X2.

In the previous section, we have looked at some models called linear programming models. In each case, the model had a function called an objective function, which was to be maximized or minimized while satisfying several conditions or constraints.

If there are only two variables, one can use a graphical method of solution. Let us begin with the set of constraints and consider them as a system of inequalities. The solution of this system of inequalities is a set of points, S. Each point of the set S is called a feasible solution. The objective function can be evaluated for different feasible solutions and the maximum or minimum values obtained.

Graph (Linear): A linear graph consists of a number of nodes or junction points, each joined to some or all of the others by arcs or lines.

3.2 STEPS FOR SOLVING GNERAL GRAPHICAL PROBLEM

The various steps for solving Graphical problems are as follows

- Formulate te problem with mathematical form by
  - Specifying the decision variables
  - Identifying the objective function
  - Writing the constraint equations
- Plot the constraint equation on a graph
- Identify the area of feasible solution
- Locate the corner points of the feasible region
- Plot the objective function
- Choose the points where objective functions have optimal values

SOLVED EXAMPLES OF GRAPHICAL PROBLEM

Example (1)

Maximize $Z = 4x + 5y$

Subject to:

$$2x + 5y \le 25$$
$$6x + 5y \le 45$$

$$x \ge 0 \, , \, y \ge 0$$

Solution:-

To solve the above linear programming model using the graphical method, we shall turn each constraints inequality to equation and set each variable equal to zero (0) to obtain two (2) coordinate points for each equation (i.e. using double intercept form). Having obtained all the coordinate points, we shall determine the range of our variables which enables us to know the appropriate scale to use for our graph. Thereafter, we shall draw the graph and join all the coordinate points with required straight line.

$2x + 5y = 25$ [Constraint 1]

When $x = 0$, $y = 5$ and when $y = 0$, $x = 12.5$.

$6x + 5y = 45$ [Constraint 2]

When $x = 0$, $y = 9$ and

when: y = 0 , x = 7.5 .
Minimum value of x is x = 0 .
Maximum value of x is x = 12.5 .
Range of x is 0 ≤ x ≤ 12.5.
Minimum value of y is y = 0 .
Maximum value of y is y = 9 .

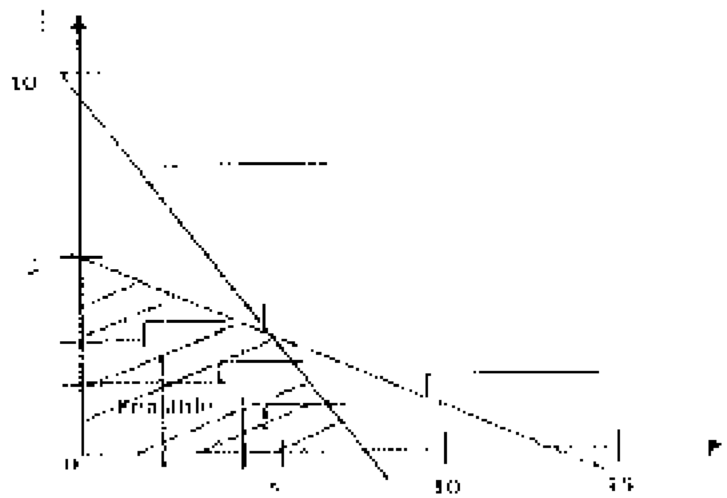

Fig.3.1

The constraints give a set of feasible solutions as graphed above. To solve the linear programming problem, we must now find the feasible solution that makes the objective function as large as possible. Some possible solutions are listed below:

	The point	Max z = 4x + 5y
A	(0,0)	$Z_A = 0$
B	(0,5)	$Z_B = 0*4+5*5= 25$
C	(5,3)	$Z_C = 5*4+3*5=35$: **is the optimal sol**
D	(7.5,0)	$Z_D = 7.5*4+0*5=30$

It turns out that (5,3)provide the maximum value: $4(5) + 5(3) = 20 + 15 = 35$.

Hence, maximum profit at point (5,3) and it is the objective functions which have optimal value.

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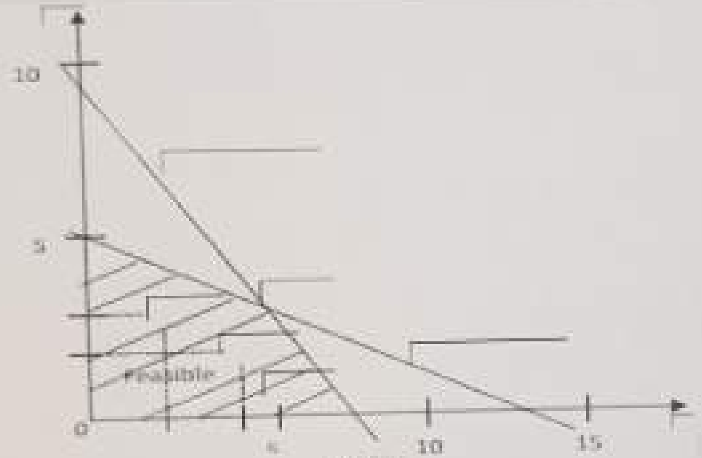
Maximum value of y is y = 9 .



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D	(7.5,0)	$Z_D = 7.5*4+0*5=30$

It turns out that (5,3)provide the maximum value: 4(5) + 5(3) = 20 +15 = 35 .

Hence, maximum profit at point (5,3) and it is the objective functions which have optimal value.

4- Technique

To illustrate this method we see the follow example:-

Minimize $Z = 3x_1 + 8x_2 + x_3$

S. to

$6x_1 + 2x_2 + 6x_3 \geq 6$

$6x_1 + 4x_2 = 12$

$2x_1 - 2x_2 \leq 2$

$x_1, x_2, x_3 \geq 0$

Sol

When put this example at standard form (1) add S_2 to constraints which less than or equal ($\leq$).

(2) - S_1 to greater than or equal ($\geq$)

(3) Add R_1, R_2 Artificial variables to the constraint which ($\geq$) or ($=$), which must be zero.

4- Penalize R_1 and R_2 in the objective function by assigning them very larg positive coefficient in objective function. Let $M > 0$ be

2nd-Iteration

BV	Z	x_1	x_2	S_1	S_2	S_3	S_4	RHS
Z	1	0	0	$\frac{1}{3}$	$\frac{4}{3}$	0	0	$38/3$
x_2	0	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	0	$4/3$
x_1	0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	0	$\frac{10}{3}$
S_3	0	0	0	-1	1	1	0	3
S_4	0	0	0	$-2/3$	$\frac{1}{3}$	0	1	$\frac{2}{3}$

Note

If all value of Z are non negative the solution is stope

The Optimal solution gives as

BV — RHS

$Z = \frac{38}{3}$

$X_2 = 4/3$

$X_1 = \frac{10}{3}$

$S_3 = 3$

$S_4 = 2/3$

Non basic

$S_1 = 0, S_2 = 0$

Checking

$Z = 3x_1 + 2x_2$

$\frac{38}{3} = 3\left(\frac{10}{3}\right) + 2\left(\frac{4}{3}\right)$

$\frac{38}{3} = \frac{38}{3}$

(9)

Z: 1 −3 −2 0 0 0 0 0
−[−3 (0 1 $\frac{1}{2}$ 0 $\frac{1}{2}$ 0 0 4)

1 0 $-\frac{1}{2}$ 0 $\frac{3}{2}$ 0 0 12

S_1 0 1 2 1 0 0 0 6
−1 [0 1 $\frac{1}{2}$ 0 $\frac{1}{2}$ 0 0 4

S_1 0 0 $\frac{3}{2}$ 1 $-\frac{1}{2}$ 0 0 2

S_3 0 −1 1 0 0 1 0 1
+1[−(0 1 $\frac{1}{2}$ 0 $\frac{1}{2}$ 0 0 4)
0 0 $3/2$ 0 $\frac{1}{2}$ 1 0 5

EV

First − iteration

BV	Z	x_1	x_2	S_1	S_2	S_3	S_4	RHS	Ratio
Z	1	0	$-1/2$	0	$3/2$	0	0	12	x
S_1	0	0	$3/2$	1	$-\frac{1}{2}$	0	0	2	$2/3/2 = \frac{4}{3}$ min
x_1	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	4	$\frac{4}{1/2} = 8$
S_3	0	0	$3/2$	0	$\frac{1}{2}$	1	0	5	$5/3/2 = 5$
S_4	0	0	1	0	0	1	2	2	2

LV / PR → S_1

By using the same above steps
Can find the following table:

Simplex

EV

BV	Z	x_1	x_2	s_1	s_2	s_3	s_4	RHS	R
Z	1	-3	-2	0	0	0	0	0	
s_1	0	1	2	1	0	0	0	6	6/1=
NPR → s_2	0	(2)	1	0	1	0	0	8	8/2 =
s_3	0	-1	1	0	0	1	0	1	x
s_4	0	0	1	0	0	0	1	2	X

pivot Element

Step 3: Entering variable (EV)

min most -ve coefficient in Z row

$\min\{-3, -2\} = -3$

Step 4: Leaving variable (L.V)

$$\text{Ratio} = \frac{\text{RHS}}{+\text{ve pivot column coefficient}}$$

$\min\{\frac{6}{1}, \frac{8}{4}\} = \{6, 4\} = 4$

$\therefore$ EV $= x_1$, LV $= s_2$

Step 5

(1) New pivot Row (NPR) $= \frac{\text{old pivot Row}}{\text{pivot Element}}$

x_1 New $= \frac{s_2}{2} = \frac{0}{2}, \frac{2}{2}, \frac{1}{2}\ \frac{0}{2}\ \frac{1}{2}\ \frac{0}{2}\ \frac{0}{2}\ \frac{8}{2}$

$X_1 = 0\ 1\ \frac{1}{2}\ 0\ \frac{1}{2}\ 0\ 0\ 4$

⑦

Max $Z = 3x_1 + 2x_2$

s.to

$x_1 + 2x_2 \le 6$

$2x_1 + x_2 \le 8$

$-x_1 + x_2 \le 1$

$x_2 \le 2$

$x_1, x_2 \ge 0$

Sol

step 1 put the system in the Standard form

$Z - 3x_1 - 2x_2 = 0$

$x_1 + 2x_2 + S_1 = 6$

$2x_1 + x_2 + S_2 = 8$

$-x_1 + x_2 + S_3 = 2$

$x_2 + S_4 = 2$

$x_1, x_2, x_3, S_1, S_2, S_3$ and $S_4 \ge 0$

where $S_1, S_2, S_3 \ge 0$ slack variables

Step 2

The initial simplex table

Basic variable (BV), which have coefficie ⊖ in equation of (S.f) in one equatio which written in initial simplex table

New Pivot Row (NPR) = $\dfrac{\text{Old pivot Row}}{\text{Pivot Element}}$

$x_{1\,New} = \dfrac{S_1 Row}{\frac{25}{8}} = 1 \quad 0 \quad 0 \quad \dfrac{8}{25} \quad \dfrac{7}{16} \times \dfrac{8}{25} \quad \dfrac{-16}{25} \quad \dfrac{78}{25}$

In same way can find the following table

BV	Z	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	1	0	0	0	$\frac{-9}{25}$	$\frac{-18}{25}$	$\frac{-4}{25}$	$-\frac{314}{25}$
x_1	0	1	0	0	$\frac{8}{25}$	$\frac{11}{50}$	$-\frac{16}{25}$	$78/25$
x_2	0							$\frac{114}{25}$
x_3	0							$11/10$

Note

If all values of Z are not positive the solution are stoped

The optimal solution gives as:

BV		RHS
Z	=	$\frac{-314}{25}$
x_1	=	$\frac{78}{25}$
x_2	=	$\frac{114}{25}$
x_3	=	$\frac{11}{10}$

في min يجب ان لا توجد قيمة موجبه
في حالة الحصول ضمنيا بأكبر قيمة موجبه
في max يجب ان لا توجد قيمة سالبة في الصف
ونبدأ بأكبر قيمة سالبة

Non basic

$S_1 = 0$

$S_2 = 0$

$S_3 = 0$

Checking

$Z = \frac{78}{25} - 3\left(\frac{114}{25}\right) - 2\left(\frac{11}{10}\right)$ [illegible]

$\frac{-314}{25} = \frac{-314}{25}$

EV (5)

2n- iteration

BV	Z	x_1	x_2	x_3	S_1	S_2	S_3	RHS	Ratio
Z	1	$\frac{1}{2}$	0	2	0	$-3/4$	0	-9	x
S_1	0	$5/2$	0	2	1	$\frac{1}{4}$	0	10	$\frac{10}{2}=5$
x_2	0	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	3	x
NPR → S_3	0	$-5/2$	0	(8)	0	$-3/4$	1	1	$\frac{1}{8}$ min

pivot Element

$$\text{New pivot Row} = \frac{\text{old pivot Row}}{\text{pivot Element}}$$

$$x_{3\,New} = \frac{S_3\,Row}{8} = \frac{-5}{16} \quad 0 \quad 1 \quad 0 \quad \frac{-3}{32} \quad 1 \quad \frac{1}{8}$$

old Z : $\frac{1}{2}$ 0 2 0 $\frac{-3}{4}$ 0 -9

$-2\left[\frac{-5}{16} \quad 0 \quad 1 \quad 0 \quad \frac{-3}{32} \quad 1 \quad \frac{1}{8}\right]$

$\frac{-5}{8}$ 0 -2 0 $-\frac{3}{16}$ 2 $-\frac{1}{4}$

New Z Row $\frac{13}{8}$ 0 0 1 $\frac{7}{16}$ -2 $\frac{39}{4}$

New S_1 Row : $\frac{25}{8}$ 0 0 1 $\frac{7}{16}$ -2 $\frac{39}{4}$

Third- iteration

EV → x_1

BV	Z	x_1	x_2	x_3	S_1	S_2	S_3	RHS	Ratio
Z	1	$\frac{13}{8}$	0	0	0	$\frac{-9}{16}$	-2	$\frac{-37}{4}$	
NPR → S_1	0	$\frac{25}{8}$	0	0	1	$7/16$	-2	$39/4$	←
x_2	0	$-1/2$	1	0	0	$\frac{1}{4}$	0	3	
x_3	0	$-5/16$	0	1	0	$-3/32$	(1)	$\frac{1}{8}$	

(4)

Step 5:

① New pivot Row $= \frac{\text{Old Pivot Row}}{\text{Pivot Element}}$
(NPR)

x_2 new $= \frac{S_2 \text{ Row}}{4} = \frac{0}{4}, \frac{-2}{4}, \frac{4}{4}, \frac{0}{4}, \frac{0}{4}, \frac{1}{4}, \frac{0}{4}, \frac{12}{4}$

$x_2 = 0 \quad -\frac{1}{2} \quad 1 \quad 0 \quad 0 \quad \frac{1}{4} \quad 0 \quad 3$

First iteration

BV	Z	x_1	x_2	x_3	S_1	S_2	S_3	RHS	Ratio
Z	1	$\frac{1}{2}$	0	2	0	$-3/4$	0	-9	
S_1	0	$5/2$	0	2	1	$+1/4$	0	10	
x_2	0	$-\frac{1}{2}$	1	0	0	$1/4$	0	3	⟸
S_3	0	$-5/2$	0	8	0	$-3/4$	1	1	

② New Row = old Row − [pivot column coefficient * NPR]

Old Z: 1 −1 3 2 0 0 0 0
− {3 [0 $-\frac{1}{2}$ 1 0 0 $\frac{1}{4}$ 0 3]}

New Z 1 $\frac{1}{2}$ 0 2 0 $-\frac{3}{4}$ 0 −9

Note

① Initial BV Z, RHS 0
min first Z −9 min

② max initial BV Z, RHS 0 max
> , > 10

Step 2 The initial simplex table
Basic variable (BV), which have coefficient (+) in equation of (S.F) in one equation which written in (L.H.S) in initial simplex table

BV	Z	x_1	x_2 (EV)	x_3	S_1	S_2	S_3	RHS	Ratio
Z	1	-1	3	2	0	0	0	0	
S_1	0	3	-1	2	1	0	0	7	
→ S_2 (PR)	0	-2	(4)	0	0	1	0	12	$\frac{12}{4}=3$ min
S_3	0	-4	3	8	0	0	1	10	$\frac{10}{3}=3.333$

pivot Element

Step 3: Entering variable (EV)

Min most +ve coefficient in Z row
$\max\{-1, 3, 2\} = 3$

Step 4: Leaving variable (L.V)

$$\text{Ratio} = \frac{\text{RHS}}{+\text{ve pivot column coefficient}}$$

• $\min = \{\frac{7}{-1}, \frac{12}{4}, \frac{10}{3}\} = \{-7, 3, 3.333\} = 3$

$EV = x_2$, $L.V = S_2$

Example

Solve the following LP model of

$$\text{Min } Z = x_1 - 3x_2 - 2x_3$$

s.to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Solution

Step1 put the system in the standard fo

$$Z - x_1 + 3x_2 + 2x_2 = 0$$
$$3x_1 - x_2 + 2x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$+4x_1 + 3x_2 + 8x_3 + s_3 = 10$$
$$x_1, x_2, x_3, s_1, s_2 \text{ and } s_3 \geq 0$$

Where $s_1, s_2, s_3 \geq 0$ slack variables

Note

if $-3x_1 + 6x_2 - 2x_3 \geq -17$

Then

$$-1\left[-3x_1 + 6x_2 - 2x_3 \geq -17\right]$$
$$\therefore 3x_1 - 6x_2 + 2x_3 \leq 17$$

CH 2

2 Simplex Method

When using this method we must see the following steps:-

Step 1: Construct the standard form

Step 2: Construct the initial simplex table

Step 3: Selected an Entering Variable (EV) from the following as

max	most -ve	coefficient in Z row
min	most +ve	" " " "

Step 4: Slect leaving variable (LV)

LV is a basic variable with minimum Ratio.

$$\text{Ratio} = \frac{\text{RHS}}{\text{+V pivot column Coefficient}}$$

Step 5: Calculate New table

$$\text{New pivot Row} = \frac{\text{old pivot Row}}{\text{pivot element}}$$

New Row = old Row - [pivot column x new pivot Row]

طريقة فوجل التقريبية (VAM)

Vogel's Approximation method

وتسمى ايضاً طريقة الكلفة الفرصة

Opportunity Cost method

حيث يتم استخدام هذه الطريقة من خلال حساب الفروقات بين اقل كلفتين في الصفوف والاعمدة

تتلخص الطريقة بالخطوات الحل التالية :-

١- تنظيم مصفوفة او جدول وتخصيص مساحة مضافة لحساب الكلفة الفرصة للصفوف والاعمدة

٢- موازنة المصفوفة او الجدول اي ان ينبغي ان يكون اجمالي الطلب (الاحتياج) مساوياً لاجمالي الطاقة التجهيزية، فاذا لم يكن كذلك يجب اضافة صف او عمود وهمي.

٣- استخراج الكلفة الفرصة (الفرق) بين ادنى كلفتين في كل صف وكل عامود. مع الاخذ بعدم الاهتمام بالصف او العامود الوهمي

٤- اختيار اعلى كلفة فرصة في (الصفوف والاعمدة)

٥- امام اعلى فرق، اختيار الخلية التي تحمل اقل تكلفة ثم اشباع طلبات الخلية من الطاقة المتوفرة

٦- الغاء العامود او الصف الذي يساوي الطلب او التجهيز صفر

٧- تكرار الخطوات السابقة.

Ex] Solve the Transportion problem by using The Vogel's approximation method

[illegible]	A	B	C	D	supply
①	6	4	13	12	500
②	11	10	4	6	700
③	4	12	9	10	800
Demand	500	200	900	600	2000 / 2200

العرض 500 + 700 + 800 = 2000 + [200] —

الطلب 500 + 200 + 900 + 600 = 2200

التوزيع / المصانع	A	B (اعلى)	C	D	supply	الكلفة الفرقية الادنى
①	6	4 (200)	13	12	~~500~~ 300	②
②	11	10	4	6	700	②
③	4	12	9	10	800	⑤
④ وهمي	0	0	0	0	200	—
Demand	500	~~200~~ 0	900	600	2200 / 2200	
الكلفة الفرقية الادنى	②	⑥ ↑	⑤	④		

*

المصانع / التوزيع	A	C	D	Supply	
1	6 (300)	13	12	~~300~~ 0	6
2	11	4	6	700	2
3	4	9	10	800	5
4 وهمي	0	0	0	200	—
Demand	~~500~~ 200	900	600		
	[illegible]	5	4		

المصانع / التوزيع	A	C	D	Supply	
2	11	4	6	700	2
3	4 (200)	9	10	~~800~~ 600	5
4 وهمي	0	0	0	200	—
Demand	~~200~~ 0	900	600		
	7	5	4		

المصانع / التوزيع	C	D	Supply		
2	4 (700 x)	6 (x)	~~700~~ 0	2	—
3	9	10 (600)	~~600~~ 0	1	1
4 وهمي	0 (200)	0	200	—	—
Demand	~~900~~ 200	~~600~~ 0			
	5	4			
	9	10			

$$TC = (4 \times 200) + (6 \times 300) + (4 \times 200) + (4 \times 700) + (10 \times 600) + (0 \times 200)$$

= [ ]

Solve the following transportation problem by using The Vogel's approximation

مراكز التسويق / المصانع	A	B	C	D	Supply
M_1	20	22	17	4	120
M_2	24	37	9	7	70
M_3	32	37	20	15	50
Demand	60	40	30	110	240 / 240

تمثل (M_1, M_2, M_3) المصانع التي توزع المبيعات الى مراكز التسويق

تمثل (A, B, C, D) مراكز الاستهلاك الى مراكز التسويق

- نلاحظ ان مجموع عامود الطاقة التجهيزية (supply) = 240
- وان مجموع صف الطاقة الاستهلاكية (Demand) = 240

∴ لا نحتاج اضافة عامود او صف وهما لموازنة المبيعات

∴ نتبع مباشرة خطوات طريقة فوجل التقريبية

وهي ايجاد الفرق بين اصغر كلفتين في كل (صف، عامود)

ومن ثم تحديد أصغر كلفة لاكبر كلفة فرصة موجودة

مراكز التسويق / المصانع	A	B (الاكبر)	C	D	Supply	كلفة فرصة اولية
M_1	20	22 / 40	17	4	~~120~~ 80	(13)
M_2	24	37 ✗	9	7	70	(2)
M_3	32	37 ✗	20	15	50	(5)
Demand	60	~~40~~ 0	30	110	240 / 240	
كلفة فرصة اولية	(4)	(15) ↑	(8)	(3)		

يتم اختيار كلفة فرصة (15) حسب خطوات فوجل الحروفية اعلاه

فيتم تحديد اصغر كلفة في العامود وهي [22]

وبالتالي يلغى العامود [B]

يلغى لأن تم تشبيع المعمل M1

	A	C	D	Supply	تكلفة وهمية ثانية
M_1	20 x	17 x	4 → 80	~~80~~ 0	(13)
M_2	24	9	7	70	(2)
M_3	32	20	15	50	(5)
Demand	60	30	~~110~~ 30	200 / 200	
تكلفة وهمية	(4)	(8)	(3)		

نحدد الفرق بين اصغر تكلفتين في كل صف وعامود

ثم نحدد اكبر تكلفة وهمية بين كل صف وعامود وهي (13) ومن ثم نحدد اصغر تكلفة في الصف [13] وهي [4]

يلغى ↓ (C)

	A	C	D	Supply	تكلفة وهمية ثانية
M_2	24	9 → 30	7	~~70~~ 40	(2)
M_3	32	20 x	15	50	(5)
Demand	60	~~30~~ 0	30	120 / 120	
تكلفة وهمية ثانية	(8)	(11)	(8)		

نلاحظ ان العامود C [11] يمثل اكبر تكلفة وهمية فنحدد اصغر تكلفة فيه وهي [9] فيلغى العامود C

يلغى (D)

	A	D	Supply	
M_2	24	7 → 30	~~40~~ 10	(17)
M_3	32	15 x	50	(17)
Demand	60	~~30~~ 0	90 / 90	
	(8)	(8)		

نلاحظ ان الصف M_2, M_3 التكلفة الوهمية اكبر فاعتبر مع ملاحظة تساويهما فناخذ اقل تكلفة في العمود (7)

	A	Supply
M_2	24 → 10	~~10~~ 0
M_3	32 → 50	~~50~~ 0
Demand	~~60~~ ~~50~~ 0	60 / 60

ناخذ اقل تكلفة وهي (24) ومن ثم ناخذ التكلفة (32) وبالتالي سيتم تفريغ الخلايا

$\therefore TC = 22\times 40 + 4\times 80 + 9\times 30 + 7\times 30 + 24\times 10 + 32\times 50 = \underline{3520}$

Ex.3 using transportation problem by using The Vogel Approximation Method.

تمثل (M1, M2, M3) المصانع التي توزع المنتجات الى مراكز التسويق (A, B, C)

مراكز التسويق / مراكز التجهيز	A	B	C	Supply
M1	1	2	6	8
M2	2	4	5	12
M3	3	1	4	8
Demand	10	10	10	28 / 30

∴ يجب موازنة مراكز التسويق مع مراكز التجهيز

مراكز التجهيز = 8 + 12 + 8 = 28 + [2] = 30

مراكز التسويق = 10 + 10 + 10 = 30

∴ يجب ان نوازن بينهما وذلك باضافة صف وهمي مقداره [2]

	A	B	C	Supply	كلفة فرصة اولية
M1	1	2	6	8	(1)
M2	2	4	5	12	(2)
(يلغى) M3	3 ✓	1 8	4 ✗	~~8~~ 0	(2) ←
M وهمي	0	0	0	2	—
Demand	10	2 ~~10~~	10		
كلفة فرصة اولية	(1)	(1)	(1)		

نلاحظ ان الصف M2 , M3 هو الكلفة الفرصية (2) اي متشابهان

نختار اقل كلفة [1] اي في الصف M3

⟹

بلغى

	A	B	C	Supply	كلفة فرصة
M1	1	2 / 2	6	~~8~~ 6	1
M2	2	4 x	5	12	2
M3	0	0 x	0	2	—
Demand	10	~~2~~ 0	10	22 / 22	
كلفة فرصة ثانية	1	2	1		

نختار العامود B ذو الكلفة 2

	A	C	Supply	كلفة فرصة ثالثة
M1 (بلغى)	1 / 6	6 x	~~6~~ 0	5
M2	2	5	12	3
M3	0	0	2	—
Demand	4 ~~10~~	10	20 / 20	
كلفة فرصة	1	1		

نختار اكبر كلفة فرصة 5 في الصف M1 ومن ثم نختار اقل كلفة بالصف وهي 1

بلغى

	A	C	Supply	كلفة فرصة 4
M2	2	5 / 10	~~12~~ 2	3
M3	0	0 x	2	—
Demand	4	~~10~~ 0	14 / 14	
كلفة فرصة 4	2	5		

نختار الكلفة الفرصة 5 في العامود C

(لان الكلفة 0 وهمية اي لا تحتسب)

	A	Supply	
M2	2 / 2	~~2~~ 0	2
M3	0 / 2	~~2~~ 0	—
Demand	~~4~~ 2	4 / 4	
	2		

(التكلفة الكلية)

$$\therefore TC = 1\times8 + 2\times2 + 1\times6 + 5\times10 + 2\times2 + 2\times0$$

$$= 72$$