الجامعة التكنولوجية
علوم الحاسوب

DIGITAL SIGNAL PROCESSOR

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Computer Science

Multimedia *Third stage*

Chapter one

1.1 Signal. System and Signal processing:

Signal: is defined as any physical quantity that Varies with time space or any other independent Variable.

EX;

S1(t)=5t one Independent Varies

S2(x,y)=3x+2xy=10y² Two Independent Varies

$S3(t)=\sum_{i=1}^{N} Ai(t)\sin[2\pi Fi(t)t+\Theta i(t)\ ]$ Segment of Speeh

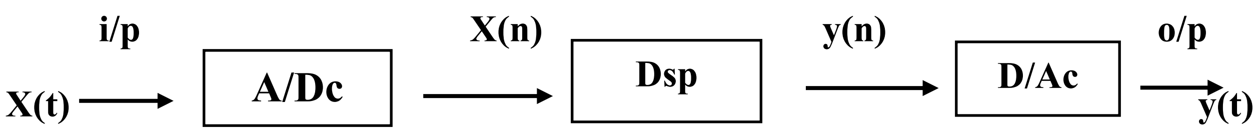
Where Ai: Amplitudes

Fi: Frequency

Ɵi: Phases

System: It is a physical device or any software that performs on Operation on a signal like filtering, multiplication, transformation amplification etc.

1.2 Basic Elements of a digital processing System:



1.3 Digital Against Analog Signal processing:

Advantages:

1. Dsp system are highly flexible.

2. Accuracy of Dsp sys. is much higher than analog Sys.

3. Dsp sys are cheaper Compared to analog Sys

4. Dsp sys are upgradable & Repeatable and small in Size

Disadvantage:

1. Dsp sys are expansive for small applications.

2. Analog signals with wide bandwidth Need to high speed of AID.

1.4 DSP applications :

1. Voice & speech [speech recognition, voice Mail]

2. Tele Communication [Cellular phone]

3. Consumer [video, Tv, Music Sys]

4. Graphic & Imaging [Animation]

5. Military [Radar, Sonar]

6.Biomedical Eng [X-ray storage and Enhancement, ECG]

1.5 Classification of Signals:

1. Multichannel and Multidimensional Signals..

2. Continuous and Discrete time signals.

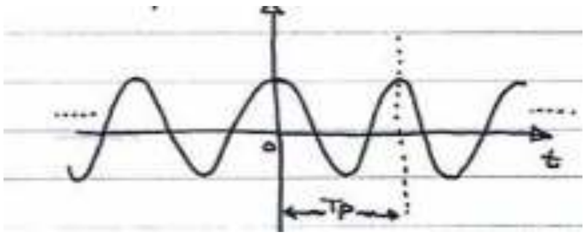
3. Continuous and Discrete Value [Amplitudes] Signals.

4. Deterministic and Random Signals.

* Continuous time Signal CTS

$$Xa(t) = Acos(\Omega t + \theta) \qquad -\infty < t < \infty \ldots\ldots.1$$

$\Omega = 2\pi F = \frac{2\pi}{T_P}$ Tp $= \frac{1}{f}$ $x_a(t)$



A. amplitude

Angular Frequency (r/s)

Ө: Phase (r)

TP: Fundamental period (s)

F: Frequency (c/s)

*Discrete time signal ΔTS

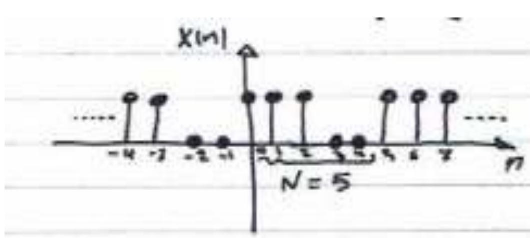
$$x_{(n)} = A\,cos(w_0 n + \theta) \qquad -\infty < n < \infty \ldots\ldots\ldots..2$$

$$w_0 = 2\pi f_0 = 2\pi \frac{k}{N}$$

$$f_0 = \frac{k}{N}$$

N; Fundamental Period

F_o: Fundamental Frequency



A discrete time signal is periodic only if Its Freq. f is :

$$X(n+N) = x(n) \qquad for\ all\ n$$

N: Smallest value of Samples which repeatable every period of time

[Fundamental Freq] $x_{(n)} = A\,cos(2\pi f \circ n + \theta)$

$$x_{(n+N)} = A\,cos(2\pi f \circ (n+N) + \theta)$$

$$= A\,cos(2\pi fon + 2\pi f \circ N + \theta)$$

For Periodicity $X(n+N) = X(n)$

$\mathrm{A\cos(2\pi f \circ n + 2\pi f \circ N + \theta)} = A\cos(2\pi f \circ n + \theta)$

This reaction is true if

$2\pi f_0 N = 2k\pi$ k:integar

$$f_0 = \frac{k}{N}$$

:: DTS is periodic only if its freq. fo is ratio of two integers.

Note1:two function x_1**(n)**+x_2(n) are periodic if :

$N_1/N_2 =$ n/m ratio of two integers

Note2: ΔTS Sinusoids whose frequencies are separated by an integer multiple by 2π are identical

$$\mathrm{x(n) = cos(w_0 n + \theta) = cos((w_0 + 2\pi)n + \theta)}$$

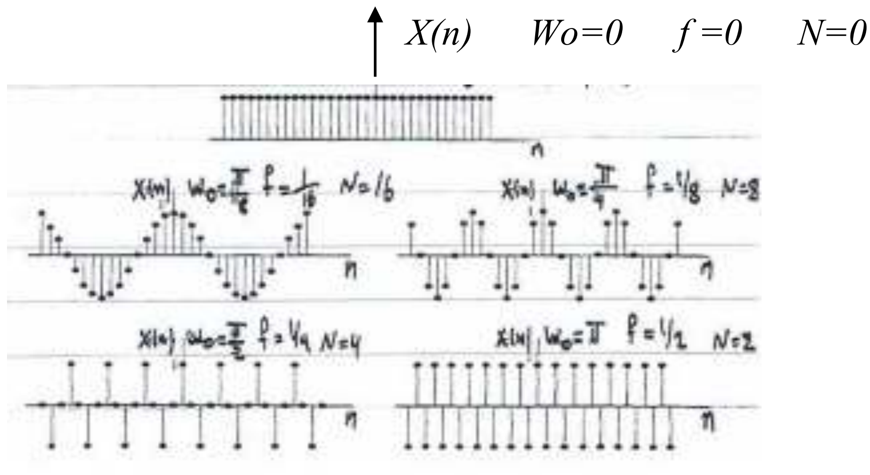
$$\mathrm{= cos(w_0 n + 2\pi n + \theta) = cos(w_0 n + \theta)}$$

$$\mathrm{:-\pi \le w \le \pi \circ r - 1/2 \le f \le 1/2}$$

Let $w_0 = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{\pi}{2}, \pi \ldots\ldots\ldots$

So $fo = 0, 1/16, 1/8, 1/4, 1/2$

$N = 0, 16, 8, 4, 2$



Ex: Determine whether the following DTS are Periodic or not? if Periodic determine fundamental Period.

1- $X(n) = cos\ 0.01\ \pi n :\ w = 2\pi f = 0.01\ \pi$

$f = \frac{0.01\pi}{2\pi} = \frac{1}{200} = \frac{k}{v}$ N = 200 samples

:period with N=200

2- $X(n) = sin3n \quad w = 2\pi f = 3$: $f = \frac{3}{2\pi}$

:Non periodic

3- $X(n) = cos\frac{n}{8} \quad cos\frac{\pi n}{8}$: $w1=2\pi f1 =1/8$

$f_1 = \frac{1/8}{2\pi} = \frac{1}{16\pi}$ Non periodic

$W2=2\pi f2 = \frac{\pi}{8}$

$F2 = \frac{\pi/8}{2\pi} = \frac{1}{16}$ periodic N=16 s

4- $X(n) = cos\frac{2\pi}{5}n + cos\frac{2\pi}{7}n$

$$w_1 = 2\pi f_1 = \frac{2\pi}{5}$$

$f_1 = \frac{2\pi}{5} * \frac{1}{2\pi} = \frac{1}{5}$ periodicN1=5

$$W2 = 2\pi f2 = 2\pi/7$$

$$f_2 = \frac{2A}{7} * \frac{1}{2\pi} = \frac{1}{7}$$

N1/N2= 5/7 ratio of two integer periodicN2=7

:X(n) is periodic to find N used LCM

N1=5 N2= 7 :LCM=35 : N=35s

Sheet 1

Q1./ Is the signals below periodic on not, find period Value?

1- *X(n)= cos3πnX(X*
2- *X(n)=sin(π+0.2n)*
3- *X(n)=¿*$^{\pi/4\ n}$
4- *X(n)= cos30π/100 n*
5- *X(n)=sin 62π/10 n*
6- *X(n)=cosn+sin*$\sqrt{2}n$

Q2/ Determine whether or Not each of the following signals is Periodic. if Periodic find fundamental Period

1-X(t)=3cos (5t+π/6)

2-X(n)=3cos (5n+π/6)

3-X(n)=2exp(j(n/6-π)

4-X(n)=cos(πn/n)-sin(πn/8)+3cos(πn/4+π/3)

5-X(t)=(cos(2πt))^2

6-X(n)=(cos(2t-π/3))^2

Q3/ The Sinusoidal Signal $X(t) = 3\cos(200t+\pi/6)$ is Passed through a square - Law device defined by the i/p-o/p relation

$$Y(t)=X^2(t)$$

using the trigonometric identity $\cos^2=\frac{1}{2}(\cos 2\Theta+1)$ show that the o/p y(t) Consists of a de Component and Sinusoidal Component

a- Specify the DC Component
b- Specify the amplitude and fundamental frequency of the Sinusoidal Component in the o/p y(t).

SAMPLING THEOREM

Chapter Two

Most Signals of Practical interest. Such as speech biological Signals, Radar Signals, Sonar Signals and Various Communications signals such as audio and video signals, are analog.

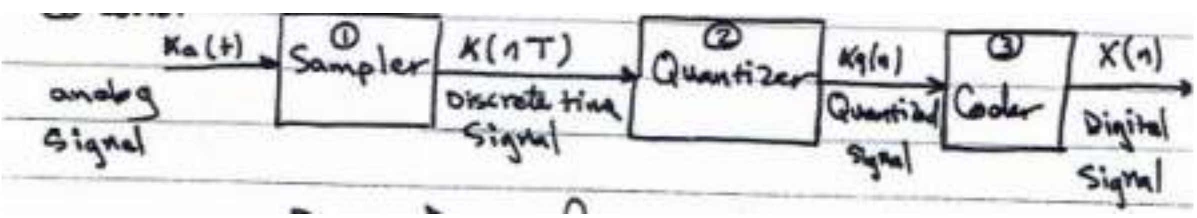
To process analog Signals by digital means, It is first necessary to Convert them into digital form. That is to Convert then to a sequence of numbers having finite precision. This procedure is Called analog-to-digital (A/D) Conversion, and the corresponding devices are called A/D Converters.

We view A/D Conversion as a three-Step process.

1-Sampler

2-Quantizer

3-Coder



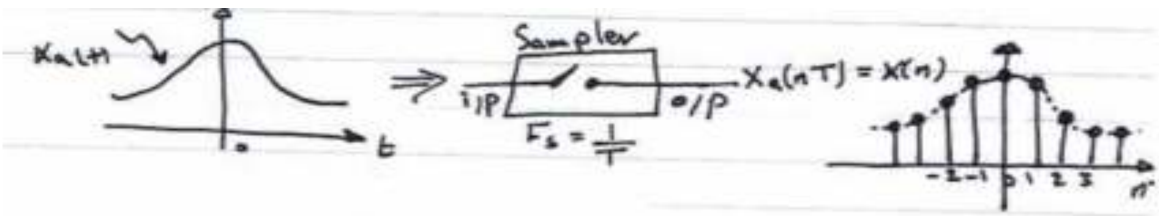
Basic parts of an analog to digital converter

2.1 sampling of analog signals:

$$: Xa(t) = Acos(\Omega t + \theta) \qquad -\infty < t < \infty \ldots\ldots\ldots 1$$

$$\Omega = 2\pi F$$

$$Xa(t) = Acos(2\pi Ft + \theta) \qquad \ldots\ldots\ldots 2$$



T: Sampling Period [Sample interval]

Fs: Sampling rate [Sampling Frequency]

$$t = n\,T = n/Fs$$

Now put 3 in 2 We get

$$Xa(nt) = X(n) = Acos\,(2\pi FnT + \theta)$$

$$= Acos\,(2\pi Fn/Fs + \theta) \ldots\ldots\ldots\ldots\ldots 4$$

If we compare 4 with

$$X(n) = Acos(2\pi fn + \theta)$$

$$: f = F/Fs \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5 \qquad w = \Omega T \ldots\ldots\ldots\ldots\ldots 6$$

:the range of frequency in CT sinusoids are

$$-\infty < \mathrm{F} < \infty$$

$$-\infty < \Omega < \infty$$

However the situation is different in ΔTS

$$-\frac{1}{2} \leq \mathrm{f} \leq \frac{1}{2}$$

$$-\pi \leq w \leq \pi \ldots\ldots\ldots 7$$

By substituting from 5 and 6 into 7 we get

$$-\frac{1}{2T} = \frac{-Fs}{2} \leq F \leq \frac{Fs}{2} = \frac{1}{2T}$$

$$\frac{-\pi}{T} = -\pi Fs \leq \Omega \leq \pi Fs = \frac{\pi}{T}$$

Note:

F=Fs/2 → Folding Frequency

CTS

$\Omega = 2\pi F$ → HZ

→ r/second

F=1/T

$-\infty < \Omega < \infty$

$-\infty < F < \infty$

ΔTS

$w = 2\pi f$ → C /sample

→ r/second

f=1/N

$$\frac{-\pi}{T} \leq \Omega \leq \frac{\pi}{T}$$

$$\frac{-Fs}{2} \leq F \leq \frac{Fs}{2}$$

CTS → ΔTS:

W=ΩT

f= F/Fs

ΔTS → CTS:

Ω=w/T

F=f*Fs

Since the highest frequency in a discrete time signal is W=π or f=½, It follows that, with a a Sampling rate Fs, the Corresponding highest values of F and Ω are

$$Fmax = Fs/2 = 1/2T$$

$$\Omega max = \pi Fs = \pi/T$$

$F_s = 2F_{MAX}$: minimum sampling rate or Nyquist rate

$Fs \geq 2F_{MAX}$: Nyquist conditions to a void aliasing

Ex1: What happens to frequencies that is not apply to Conditions. [Fs ≥ 2 Fmax] for two signals below that are Sampled at the rate of Fs = 40Hz

$$X1(t) = Cos\ 2\pi\ (10)\ t\ , X2(t) = Cos\ 2\pi\ (5dt)$$

Sol:

$$X1(t): \Omega 1max = 2\pi F1max = 2 * 10 * \pi$$

$$F1max = 10HZ$$

$$X2(t): \Omega 2max = 2\pi F2max = 2 * 50 * \pi$$

$$F2max = 50HZ$$

$$\therefore X1(n) = cos2\pi(F/Fs)n = cos2\pi(10/40)n = cos\pi/2n$$

$$X2(n) = cos2\pi(F/Fs)n = cos2\pi(50/40)n = cos5\pi/2\ n$$

$$= cos(2\pi n + \pi/2\ n)$$

$$\therefore X1(n) = X2(n) \quad = cos\ \pi/2n$$

$$\therefore F2 = 50HZ\ is\ an\ alias\ of\ F1 = 10HZ\ at\ Fs = 40HZ\ also\ at\ F3 = 90\ HZ\ , F4 = 130\ HZ$$

Ex2: Consider the analog signal

$$Xa(t) = 3\,Cos\,100\pi\,t$$

a. Determine the minimum Sampling rate required to a void aliasing.

b. Suppose that the signal is sampled at the rate Fs = 200 Hz, what is The ΔTS obtained after Sampling.

c. Suppose that the signal is Sampled at the rate Fs = 75 Hz, what is the ΔT Signal obtained after Sampling.

d. What is the frequency o <F< Fs/2 of a sinusoid that yields Samples identical to those obtained in Part C.

Sol:

a- $\Omega\,max = 2\pi Fmax = 100\pi\,, Fmax = 100\pi/2\pi = 50HZ$

$$:Fs = 2Fmax = 2 * 50 = 100HZ$$

$$\boldsymbol{b} - Fs = 200HZ$$

$$Xa(t) = 3cos100\pi t$$

$$X(n) = 3cos100\pi nT$$

$$= 3cos\,100\pi n/Fs = 3cos\,100\pi n/200 = 3cos\pi/2n$$

$$\boldsymbol{c} - Fs = 75HZ$$

$$X(n0 = 3cos100/75\pi n = 3cos4\pi/3n$$

$$= 3cos(2\pi - 2\pi/3)n = 3cos(2\pi/3)n$$

$$\boldsymbol{d} - X(n) = 3cos\,2\pi/3n$$

$$w = 2\pi f = 2\pi/3\,, f = 1/3 = Fmax/Fs : Fmax = 75/3 = 25HZ$$

$$:ya(t) = Acos\Omega t = 3cos2\pi Ft$$

$$= 3cos2\pi * 25t$$

$$Ya(t) = 3cos50\pi t : F = 50HZ\ is\ an\ alias\ of\ 25HZ$$

Ex3: consider the analog signal

$$Xa(t) = 3cos50\pi t + 10sin300\pi t - cos100\pi t$$

What is the Nyquist rate for this signal?

Sol:

$$\Omega 1 = 2\pi F1 = 50\pi \ , F1 = 50/2 \ = 25HZ$$

$$\Omega 2 = 2\pi F2 = 300\pi \ , F2 = 300/2 \ = 150HZ$$

$$\Omega 3 = 2\pi F3 = 100\pi \ , F2 = 100/2 \ = 50HZ$$

$$: Fmax = F2 = 150\ HZ$$

$$Thus\ Fs = 2Fmax \ = 300\ HZ$$

Discussion: It should be observed that the signal Component 10 Sin 300 πt, Sampled at the Nyquist rate 300 Hz Results in the Samples lo sin πn, which are identically Zero. In other word we are Sampling the analog Sinusoid at Its Zero-Crossing points, and hence. We miss this signal Component completely. This. Situation would not occur if the sinusoid is offset in phase by Some amount ϴ

$: 10\ Sin\ (300\pi t \ + \ \theta)$

Sheet 2

Q1/ Consider the analog signal

$$Xa(t) = 3\,Cos\,2000\ \pi t + 5\,sin\,6000\ \pi t + 10\,Cos\,12000\ \pi t$$

(a) what is the Nyquist rate for this signal?

(b) Assume now that we sample this signal using a Sampling rate Fs = 5000 Samples/s. what is the discrete-time signal obtained after sampling?

(C) what is the Analog signal Ya(t) We can reconstruct from the Samples if we use ideal interpolation?

Q2/ Consider the following analog sinusoidal signal

$$Xa\ (t) = 3\,Sin\ (100\pi\ t)$$

(a) sketch the signal Xa(t) for o≤t≤30ms

(b) The signal Ka (t) is sampled with a sampling rate Fs = 300 Samples/s

• Determine the frequency of the discrete-time signal x(n) = xa (nT) ,T=1/Fs. and show that it is periodic.

(C) Compute the sample values in one period of X(n). Sketch X(n) on the same diagram with xa (t). What is the period of the discrete-time signal in milliseconds?

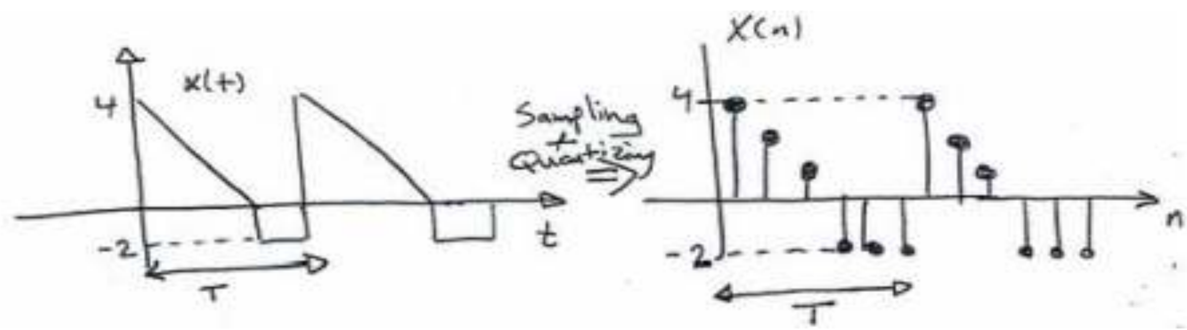
(d) Can you find a sampling rate Fs such that the signal x(n) reaches its peak value of 3? what is the Minimum Fs Suitable for this task?

Q3/ Consider the simple signal processing system shown in Fig below. The Sampling periods of the A/D and D/A Converters are T=5ms and T' = 1ms respectively. Determine the output ya(t) of the system if the input is

$$Xa(t) = 3cos100\pi t + 2sin250\pi t(\, t \; in \; seconds)$$

The postfilter removes any frequency component above Fs/2.

Q4/ for the show signal



Find the required sampling frequency Fs, the signal Completes two periods in 1msec

Q5/An analog signal Xa(t) = Sin (480πt) +3 Sin (720πt) is Sampled boo times per second

a-Determine the Nyquist Sampling rate for *Xa(t)*

b-Determine the folding Frequency

c-what are the frequencies in radios in the resulting discrete time signal x(n)?

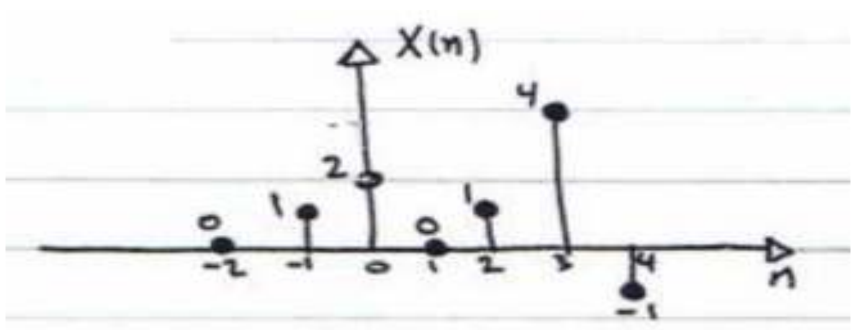
d-if x(n) is passed through an ideal D/A Converter what is the Constructed signal *ya(t)*?

DISCRETE TIME SIGNAL & SYSTEM.

Chapter Three

3.1 Discrete Time Signals:

a ΔTS X(n) is a function of an independent variable that is an integer.



Besides G.R. of ΔTS, there are some alternative representations.

* Functional Representation F.R.

Xn=	1	for	n=-1,2
	-1		n=4
	2		n=0
	4		n=3
	0		[illegible]

* Tabular representation T.R.

n	…….	-2	-1	0	1	2	3	4	…..
X(n)	…….	0	1	2	0	1	4	-1	……

* Sequence Representation S.R.

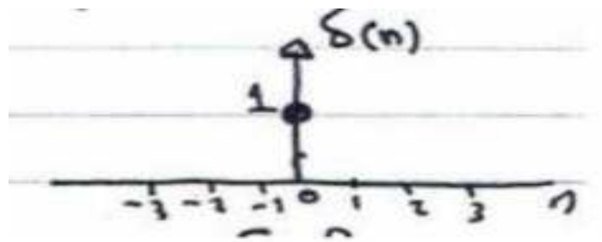
X(n)=(0,1,2,0,1,4,-1)

n=0

3.2 Some Elementary Discrete time Signals:

① Unit Sample [Unit Impulse] S(n)

F.R. S(n)= { 1 n=0 ; 0 n≠0



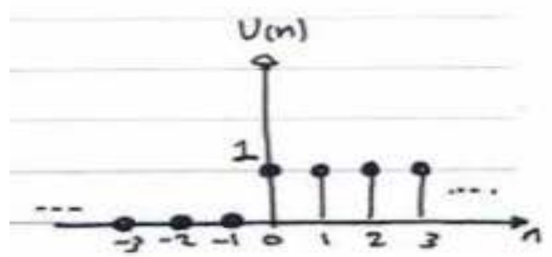
T.R.

n	…….	-2	-1	0	1	2	…..
X(n)	…….	0	0	1	0	0	…..

S.R. S(n)=(1) or (….,0,1,0,….)

2- unit up U(n)

F.R. U(n) = { 1 n= ≥ 0 ; 0 n< 0



T.R.

n	….	-2	-1	0	1	2	…..
X(n)	….	0	0	1	1	1	…..

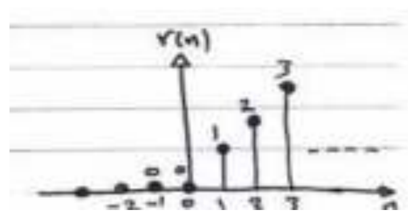
S.R. U(n)= (1,1,1,….)

Note: S(n) =u(n)-u(n-1)

U(n)= $\sum_{k=-\infty}^{n}$. S(k) or u(n)$\sum\sum_{k=0}^{\infty}$

3-unit ramp r(n) or Ur(n)

F.R. r(n)= $\begin{cases} n & n\geq 0 \\ 0 & n< 0 \end{cases}$



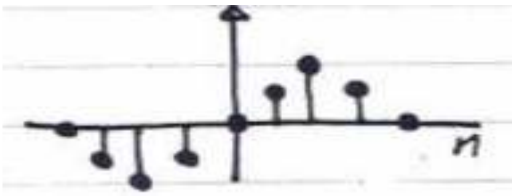
T.R.

n	….	-2	-1	0	1	2	3	.
X(n)	….	0	0	0	1	2	3	.

S.R. r(n) =(0,1,2,3,…..)

4-sinusoidal

X(n)=sin Wn

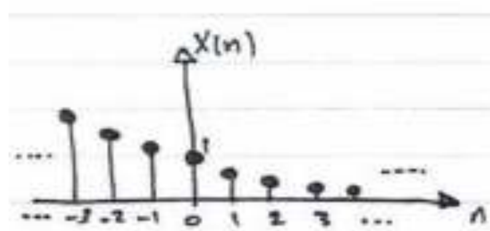


x(n)=cos Wn

5-Exponential e^{an} or a^n

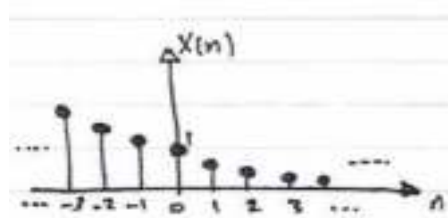
a<0 X(n)= e^{an}

a<1 X(n)= a^n



a>0 x(n)=e^{an}

a> 1 x(n)= a^n



3.3 Classification of DTS

1- Energy Signals & Power Signals The Energy of the ΔTS is denoted by E, Iris given as

$E=\sum_{n=-\infty}^{\infty}\left|x_{(n)}\right|^2$ in Joule

The Signal X(n) is Called Energy Signal if Its energy is finite (i.e all Non periodic Signals are Energy signal). Many Signals that possess infinite energy have a finite average Power. The average Power of a discrete time signal x(n) is defined as

$P=\lim N \to \infty$ 1/2N+1 $\sum_{n=-N}^{N}\left|x_{(n)}\right|^2$ in watt

Note: if x(n) is a periodic signal with fundamental period N and takes on finite values, Its Power is given by

$P=1/N \sum_{n=0}^{N-1}\left|x_{(n)}\right|^2$

For non periodic signal (Energy signal) E= finite p= zero

For periodic signal (power signal)E=∞ P= finite

Ex :Determine whether the following Signals are energy or power signals and Calculate their E or P?

1-X(n)=(1/2)^n u(n)

2-X9n0=u(n)

Sol:

1-X(n)= (1/2)^n u(n) ⟶ not periodic, it is Energy signal

$E=\sum_{n=-\infty}^{\infty}\left|x_{(n)}\right|^2 = \sum_{n=0}^{\infty}\left[(\frac{1}{2})\hat{}n\right]\hat{}2 = \sum_{n=0}^{\infty}(1/4)\hat{}n$

E=(1/4)^+(1/4)^1+(1/4)^2+(1/4)^3+……

E=1+1/4+(1/4)^2+(1/4)^3+…..

$E=\frac{1}{1-\frac{1}{4}}$ = 4/3 joule

Note: The infinite geometric series given as

1+A+A^2+A^3+….=1/(1-A) if $|A| < 1$

2-X(n)=u(n) ⟶ periodic , it is power signal

$P=\lim_{n=\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x_{(n)}|^2.=\lim_{n=\infty}\frac{1}{2N+1}\ \sum_{n=0}^{N}(1)^\wedge2$

$:\sum_{N=\infty}^{N}(1)^\wedge2$ =1+1+1+…. ,Note:1+1+1+….(N+1)=(N+1)

$:P=\lim N \rightarrow \infty\ \frac{1}{2N+1}.(N+1)$

$=\lim N \rightarrow \infty\frac{1+\frac{1}{N}}{2+\frac{1}{N}}$ =1/2 watt

2-Periodic and Non Periodic Signals any signal is periodic if x(n+N)=x(n)

X(n) = A Sin 2 π fo n ⟶ fo=k/N must rationally Function

3-Symmetric [Even] and antisymmetric [Odd] signals.

Even signal:- A signal is said to be even signal if inversion of time axis does not change the Amplitude. X(n) = x(-n)

odd Signal: A signal is Said to be odd signal if inversion of time axis also invert Amplitude of signal. X(n) = -x(-n)

Even part :Xe(n)=1/2(X(n)+X(-n))= X(n)+X(-n)/2

Odd part :Xo(n)=1/2(X(n)-X(-n))= X(n)-X(-n)/2

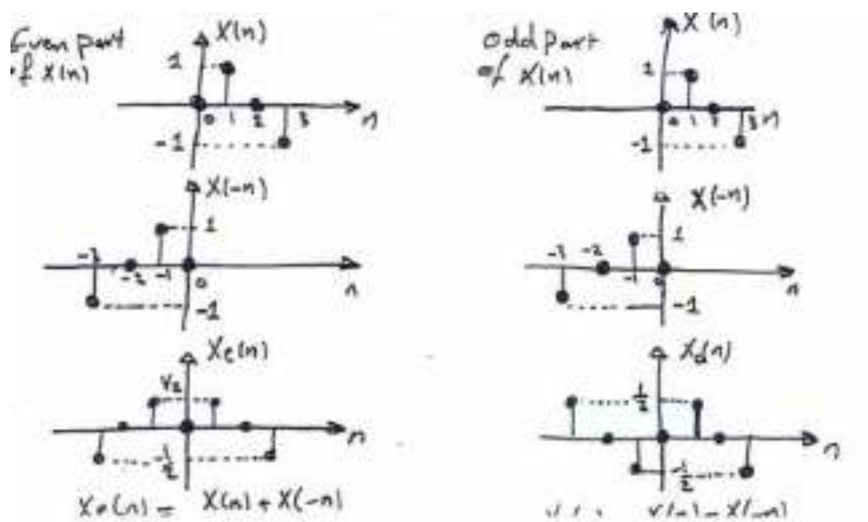
Ex: Find and sketch the Even & odd parts of the signal.

1-X(n)=sin2πn/4 U(n)

2-X(n)=$e^{-(\frac{n}{4})}$ u(n) .H.W

1- X(n)=sin2πn/4 U(n)

w=2πf =π/2 ,f=1/4 =K/N ,N=4 samples



3.4 Discrete Time System:

is a device or algorithm that performs some prescribed operation on the discrete time signal.

operation on the discrete time signal .

y(n)=T(X(n))….D.E difference equation

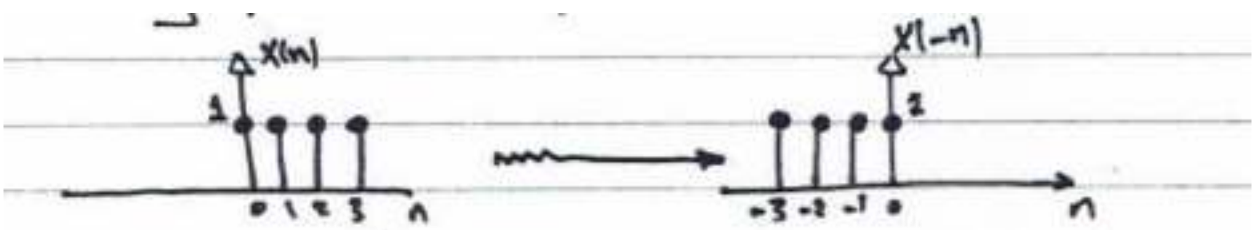
EX: y(n)=X(n)-X(n-1)+y(n-1)

X(n) $\xrightarrow{T}$ *y(n)* *X(n)=i/p*

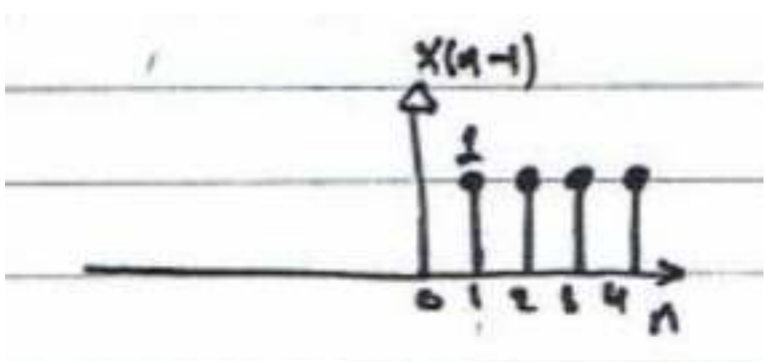
Y(n)=o/p, Response

These operation are:

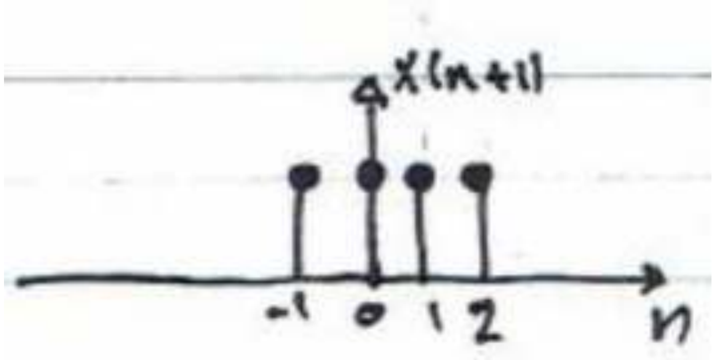
1-folding operation $\boldsymbol{y(n) = x(-n)}$



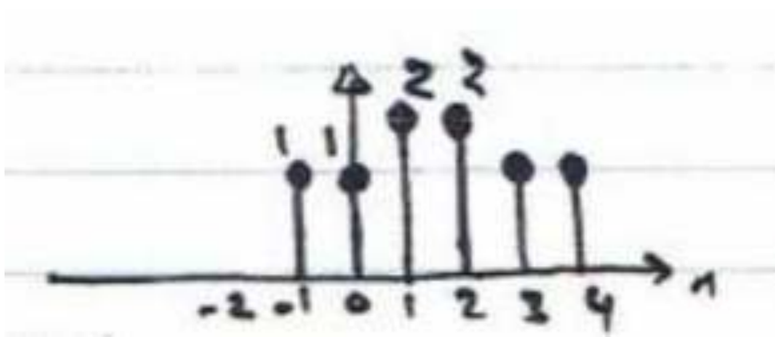
2-Delay operation $\boldsymbol{y(n) = x(n-1)}$



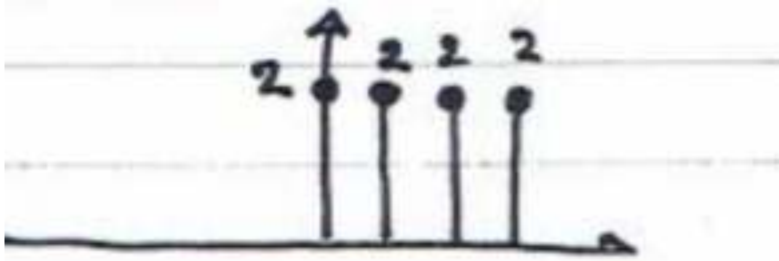
3-Advance operation $y(n) = x(n+1)$



4-Adding operation $y(n) = x(n-1) + x(n+1)$



5- Scaling operation $y(n) = 2x(n)$



Note: It is important to note that the operations of folding and time delaying or advancing a signal are not Commutative

Let Delay operation ⟶ TD

Folding operation ⟶ FD

TDk (X(n)) =X(n-k) , k>0

FD (X(n)) = X(-n)

Now TDk (FD(X(n)) = TDk (X(-n)) =X(-n+k)

Whereas FD (TDk(X(n)) =FD (X(n-k)) =X(-n-k)

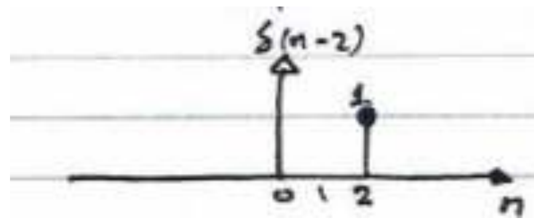
6-time scaling ⟶ compression X(mn)

⟶ Stretching X(n/m)

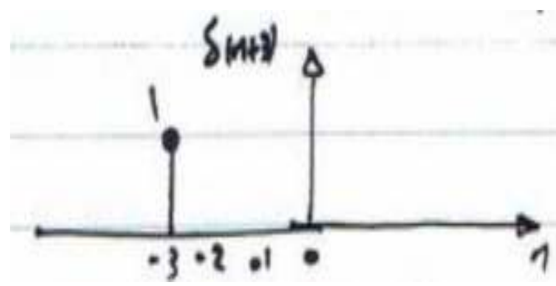
M:integer

Ex 1:sketch the following signals and system

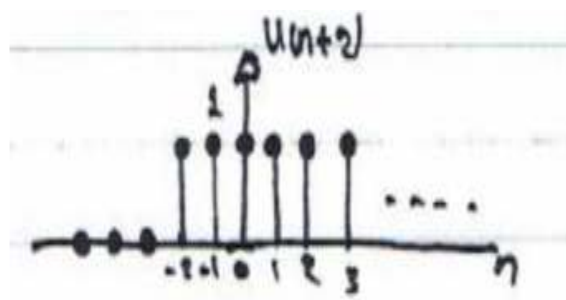
1- S(n-2)



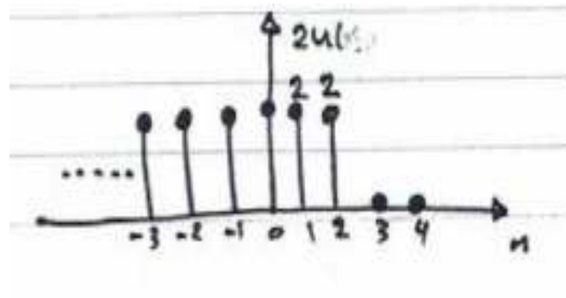
2-S(n+3)



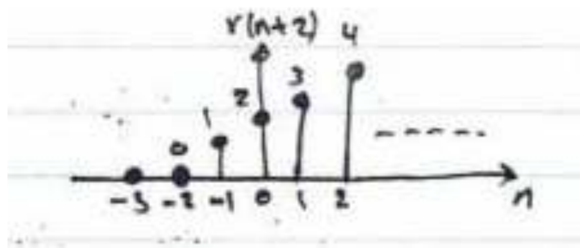
3-U(n+2)



4-2u(-n+2)



5-r(n+2)

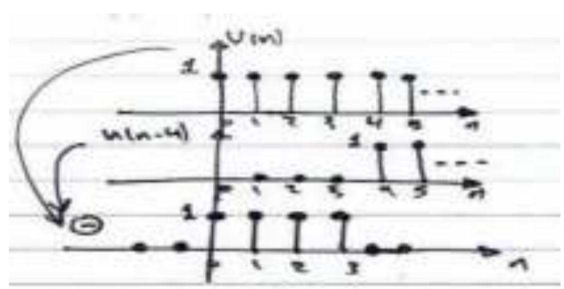


7- x(n) = u(n)-u(n-4)

write x(n) using s(n) only

sol:

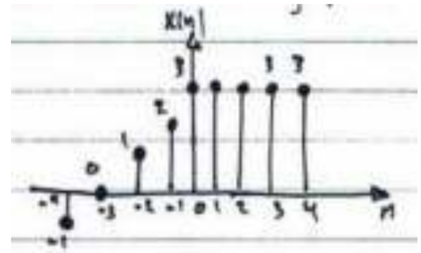
X(n)=s(n)+s(n-1)+s(n-2)+s(n+3)



Ex2: if the signal X(n) as show below find the graphical representation y(n)

y1(n)=X(2n)

y2(n)=X(n/2)



sol:

X(2n)

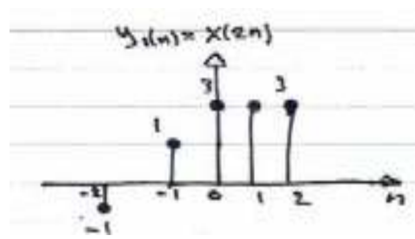
n=-2 x(2n)=x(-4)=-1

n=-1 x(2n)=x(-2)=1

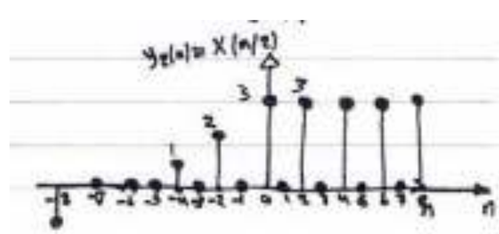
n=0 x(2n)=x(0)=3

n=1 x(2n)=x(2)=3

n=2 x(2n)=x(4)=3



X(n/2)	X
n=0	x(0) =3
n=1	x(1/2)=0
n=2	x(1)=3
n=4	x(2)=3



Ex3: A discrete time signal X(n) is detined as follow

$$X(n) = \begin{cases} 1+n/3 & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{else where} \end{cases}$$

a. Determine Its value then find G.R. & S.R.

b. Sketch the signals that if we

1. First Fold xin) and then delay the resulting signal. by Four Samples.
2. First delay X(n) by Four Samples then Fold the resulting
3. Is the relationship between 1/2 is Commutative or Not.

Sol:

a- 1+n/3 -3≤ n≤ -1

n=-3 x(-3)= 1+-3/3=0

n=-2 x(-2) =1+-2/3=1/3

n=-1 x(-1) =1+-1/3 =2/3

S.R X(n)=(0,1/3,2/3,1,1,1,1)

b- 1) X(n) $\xrightarrow{\text{Fold}}$ X(-n)

S.R X(n)=(1,1,1,1,2/3,1/3,0)

X(-n) $\xrightarrow{\text{Delay by 4}}$ X(-n+4)

S.R X(-n+4) =(0,1,1,1,1,2/3,1/3,0)

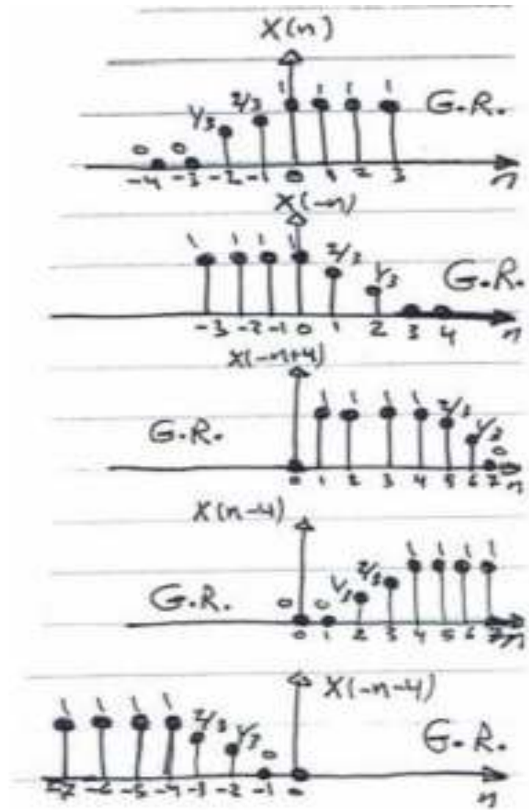
2)- X(n) $\xrightarrow{\text{Delay by 4}}$ X(n-4)

S.R X(n-4) =(0,0,1/3,2/3,1,1,1,1)

X(n-4) $\xrightarrow{\text{Fold}}$ X(-n-4)

S.R X(-n-4)=(1,1,1,1,2/3,1/3,0,0)

3)- not commutative X(-n+4)≠X(-n-4)



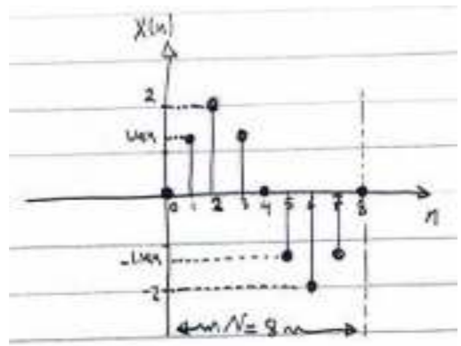
Ex4: find the number of samples cycle for the given signals then sketch

1- $X(n) = 2sin\pi/4n\, u(n)$

Sol:

$$W = 2\pi f \;= \pi/4 \;\; f = \pi/4 * 1/2\pi \;= 1/8 = k/N \;= N = 8\, s/c$$

n	X(n)
0	0
1	1.414
2	2
3	1.414
4	0
5	-1.414
6	-2
7	-1.414
8	0



2- $X(n)=e^{n/15} \sin n\pi/6\, u(n)$

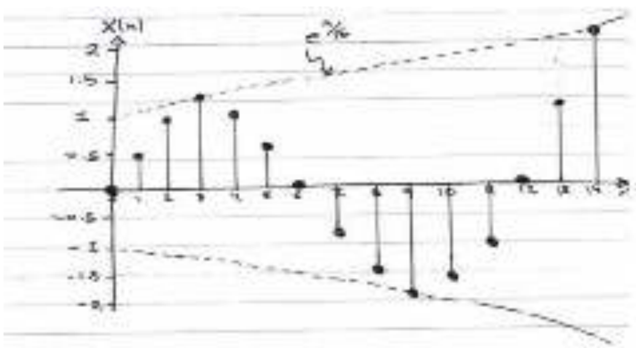
Sol:

$$W = 2\pi f \;=\; \pi/6\,, f \;=\; \pi/6 *\; 1/2\pi \;= 1/12 \;= k/N\,, \qquad N = 12\, s/c$$

$e^{n/15}$,, 1/15>0 : increasing

n	X(n)
0	0
1	0.5
2	0.9
3	1.2
4	1.1

5	0.6
6	0
7	-0.7
8	-1.5
9	-1.8
10	-1.6
11	-1
12	0
13	1.18
14	2.2



3.5 Blok diagram Representation of DT system:

Any discrete time System Cam be represented in Block diagram form. For this purpose, we need to define Some basic building blocks that Can be interconnected to form Complex Systems.

1- An adder $\sum$ or + X1(n) → (+) → **y(n)=x1(n)+x2(n)**

X2(n) → (+)

2- A constant Multiplier

X(n) → ax1(n) y(n)=ax(n)

3- Aunit delay element (Z^{-1})

$X(n) \longrightarrow Z^{-1} \longrightarrow X(n-1)$ $y(n)=x(n-1)$

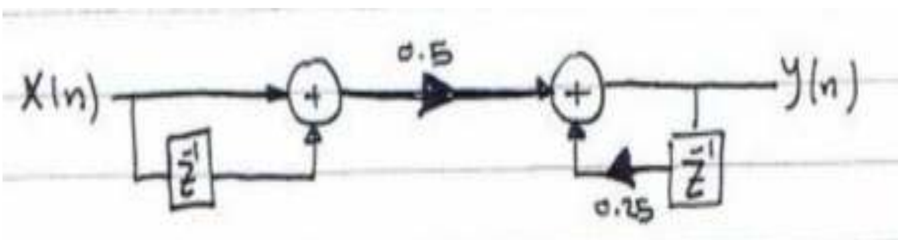
4-Avnit advance element Z^1

$X(n) \longrightarrow Z^{1} \longrightarrow X(n+1)$ $y(n)=X(n+1)$

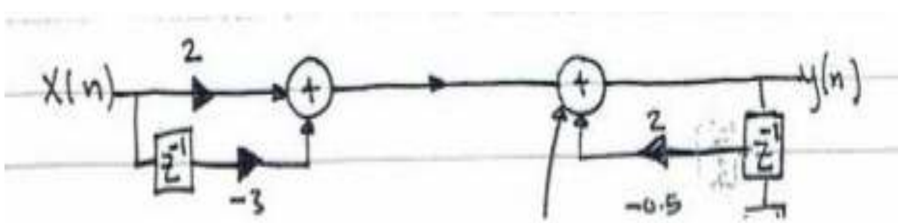
Ex1: Using basic building blocks introduced above, sketch the block diagram representation of the ΔT system described by i/p. o/p relation:

1-y(n)=1/4 y(n-1)+1/2 x(n)+1/2 x(n-1) where x(n) is i/p if y(n) o/p

Sol:



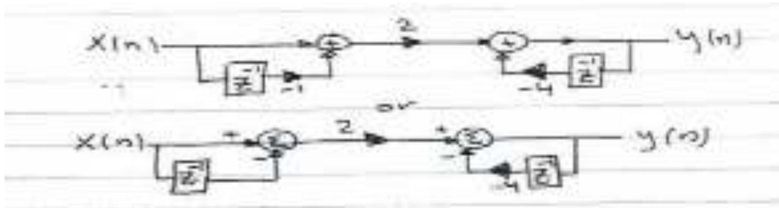
2-*y(n)=2x(n)-3x(n-1)+2y(n-1)-0.5y(n-2)*



3-*2y(n-1)+0.5y(n)=x(n)-x(n-1)*

(0.5y(n)=x(n)-x(n-1)-2y(n-1))/0.5

Y(n)=2x(n)-2x(n-1)-4y(n-1)

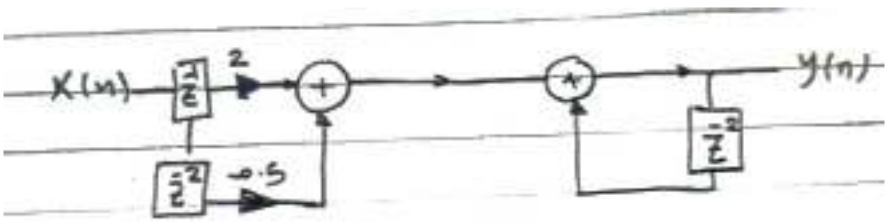


4-$y(n+1)-2x(n)+0.5x(n-2)-y(n-1)=0$

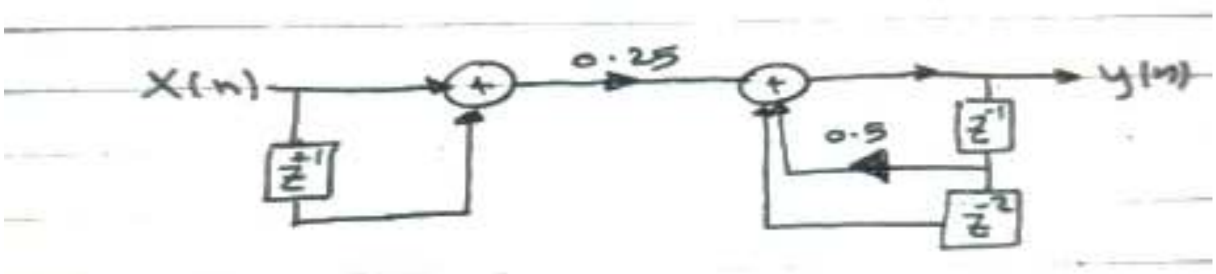
$Y(n+1-1)-2x(n-1)+0.5x(n-2-1)-y(n-1-1)=0$

$Y(n)-2x(n-1)+0.5x(n-3)-y(n-2)=0$

$Y(n)=2x(n-1)-0.5x(n-3)+y(n-2)$



Ex_2: find the difference equation from the block diagram bellow



Sol:

Y(n)=0.25x(n)+0.25x(n+1)+0.5y(n-1)+0.5y(n-3)

3.6 Classification of Discrete Time Systems:

DT system may be classified in terms of the properties that they Possess.

Note: *y(n)=x(n) n=0 , y(0)=x(0)

n=1 ,y(1)=x(1)

n=-1 ,y(-1)=x(-1)

} this o/p depends on present i/p

*y(n)=x(n-1) n=0 , y(0)=x(-1)

n=1 , y(1)=x(0)

n=-1 , y(-1)=x(-2)

} this o/p depends on past i/p

*y(n)=x(n+1) n=0 ,y(0)=x(1)

n=1 ,y(1)=x(2)

n=-1 ,y(-1)=x(0)

} this o/p depends on future i/p

① Static and Dynamic Systems:-

When the o/p depends only upon present i'p, then the system Called Static System [memoryless]. On the other hand if off depends upon Pastor Future isp, System Said to be Dynamic System [Memory]

Ex1: Is the System below Static or dynamic System.

*y(n)=10x(n) n=0 , y(0)=10x(0)

n=1 , y(1)=10x(1)

n=-1 , y(-1)=10x(-1) : static system or memory less system

*y(n)=3x(n)-4x(n-1) n=0 , y(0)=3x(0)-4x(-1)

n=1 , y(1)=3x(1)-4x(0)

n=-1 , y(-1)=3x(-1)-4x(-2) :dynamic sys. or memory sys.

② Time In variant and Time Variant System:-

If the i/p 4 o/p characteristics of the System do not change with shift of time origin, Such System Called Shift Invariant Sys. or Time Invariant Sys, otherwise Called Time Variant Sys.

Ex2: Determine whether the following Systems are shift Invariant or Shift Variant

*$y(n)=x(n)-x(n-1)$

$Y(n)=T(x(n))=x(n)-x(n-1)$

Let us shift the i/p by (k) samples. then the o/p will be $y(n,k)=T(x(n-k))=x(n-k)-x(n-k-1)$

Now let us shift the o/p by (k)

$Y(n-k)=x(n-k)-x(n-k-1)$

$:y(n,k)=y(n-k)$ *:shift invariant*

*$y(n)=nx(n)$

$Y(n)=T(x(n))=nx(n)$

Let us shift i/p by (k). o/p is

$Y(n,k)=T(x(n-k))=nx(n-k)$

Let us shift o/p by(k)

$Y(n-k)=(n-k)x(n-k)$

$:y(n,k)\neq y(n-k)$ *:shift variant*

③linear and non linear system:

A system is said to be linear if it satisfies the super-position principle

$T(x1(n)+x2(n))=T(x1(n))+T(x2(n))$

Ex3: Detemine the following system are linear or not

*$y(n)=x(n^2)$

Sol:

Let $x1(n)$ is i/p : $y1(n)=x1(n^2)$

Let $x2(n)$ is i/p : $y2(n)=x2(n^2)$

$Y(n)=y1(n)+y2(n)=x1(n^2)+x2(n^2)$

Let $x(n)=x1(n)+x2(n)$

$:y(n)=x1(n^2)+x2(n^2)$ *: linear*

*$y(n)=x(n)$

Sol: let $x1(n)$ is i/p : $y1(n)=x1^2(n)$

let $x2(n)$ is i/p : $y2(n)=x2^2(n)$

$y(n)=y1(n)+y2(n)=x1^2(n)+x2^2(n)$

let $x(n)=x1(n)+x2(n)$

$:y(n)=(x1(n)+x2(n))^2=x1^2(n)+x1(n)x2(n)+x2^2(n)$

$\neq y1(n)+y2(n)$ *;non linear*

4-Causal and Non Causal Systems:-

If o/p depends upon Past and present isp the sys is Causal. If o/p depends upon future i/p the sys is Non Causal

Ex4: check the systems which are described by D.E are Causal or Non Causal.

*$y(n)=x(n)+x(n-1)$:sys is causal

present past

*$y(n)=x(n)+x(n-1)+x(n+2)$:sys is non causal

future

⑤ Stable and unstable Systems:-

When every bounded iyp produces abounded o/p the system Called Stable [BIBO].

Ex5: is the system bellow stable or not

*$y(n)=y^2(n-1)+x(n)$

Let $x(n)=u(n)+y(-1)=0$

$Y(n)=y^2(n-1)+u(n)$

$n=0$, $y(0)=y^2(-1)+u(0)=1$

$n=1$, $y(1)=y^2(0)+u(1)=2$

$n=2$, $y(2)=y^2(1)+u(2)=5$

;o/p is unbounded the sys is unstable

Ex6: check the DT system : bellow whether these system are:

1- static or dynamic
2- linear or not
3- shift invariant or not

y(n)=3x(n)-4x(n-1)

n=0 , y(0)=3x(0)-4x(-1)

n=1 , y(1)=3x(1)-4x(0) → past : dynamic

-let i/p is x1(n)

Y1(n)=3x1(n)-4x1(n-1)

Let i/p is x2(n)

Y2(n)=3x2(n)-4x2(n-1)

Y(n)=y1(n)+y2(n)=3x1(n)+3x2(n)-4x(n-1)-4x2(n-1)

Let x(n)=x1(n)+x2(n)

Y(n)=3(x1(n)+x2(n))-4(x1(n-1)+x2(n-1)) :linear

-y(n)=T(x(n))=3x(n)-4x(n-1)

Let shift i/p by (k) then o/p is

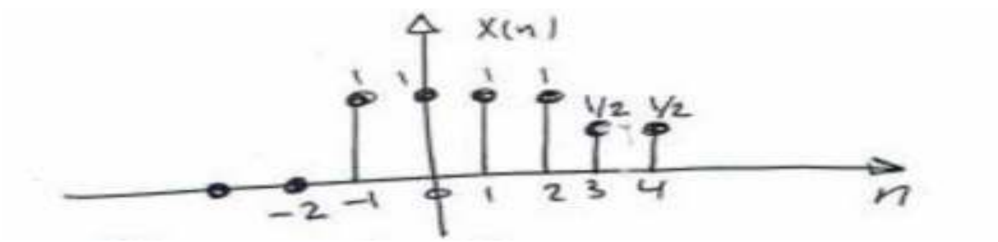
Y(n,k)=T(x(n-k))=3x(n-k)-4x(n-k-1)

Let shift o/p by (k) then o/p is

Y(n-k)=3x(n-k-4x(n-k-1):y(n,k)=y(n-k):shift invariant

Sheet 3

Q1 / A discrete - time signal X(n) is shown below. sketch and Label carefully each of the following signals.



a- x(n-2) *e-x(n-1)s(n-3)*

b- x(4-n) *f-x(n^2)*

c- x(n+2) *g-even part of x(n)*

d- x(n) u(2-n) *h-odd part of x(n)*

Q2/ Consider the system $y(n)=T[x(n)] = x(n^2)$

assume that the input signal is:

$$x(n)=\begin{cases} 1 & 0\leq n\leq 3 \\ 0 & \text{else where} \end{cases}$$

① sketch the signal X(n)

② Determine and sketch the signal y(n) = T[x(n)]

③ sketch the signal y'2 (n) = y(n-2)

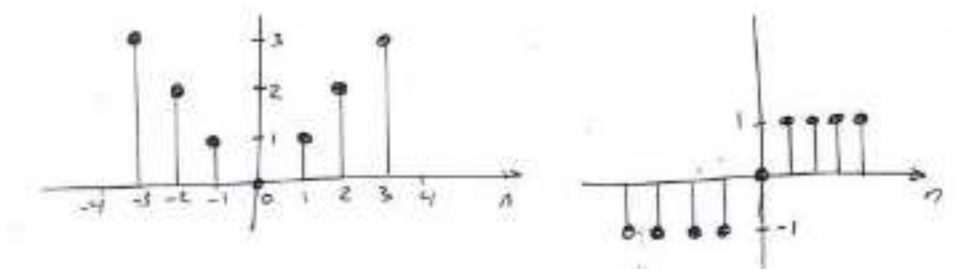
④ Determine and sketch the signal X2(n) = x(n-2)

⑤ Determine and sketch the signal $y_2(1) = T[x_2(n)]$

Q3/ Let x(n) and y(n) be give in Figure below Carefully. sketch the following

a-*X(2n)* ***b-****x(3n-1)* ***c-****y(1-n)*

d-*4(2-2n)* ***e-*** *X(n-2)+y(n+2)* ***f-****x(2n)+y(n-4)*



Q4/ Find the Number of samples /cycle [N] of given Signals then sketch them.

a) $x(n) = 3\ Cos\ (2\pi/5n)\ u(n)$

b) $x(n) = e^{-n/15}\ sin\ n\pi/2\ \ \ u(n)$

Q5/ Find the Energy and Power of

$x(n) = 1+\cos(\pi n/2)$ Hint :

$2\cos^2(x)=1+\cos(2x)$

$2\sin^2(x)=1-\cos(2x)$

Ans: 16/9 watt

Q6/ Find the period N and sketch the following Signals

A-$X(n) = 12\ Sin\ 2\pi/7n$

B-$x(n) = \sin\ (\pi/4n+180°)$

C-$x(n) = e^{-n}\cos\ (\pi/6n-90°)$

D-$x(n) = 2^n\sin\ (\pi/3n)\ u(n)$

E-$x(n) = \sin^2\ (\pi/4n)$

F-$x(n) = \cos^2\ (\pi/5n) – ½ + ½\cos(2\ \pi/5n)$

Q7/ Determine the response of the following Systems to input signal

$$X(n)=\begin{cases} |n| & -3\leq n\leq 3 \\ 0 & \text{other wise} \end{cases}$$

a- $y(n) = x(n)$
b- $y(n) = x(n-1)$
c- $y(n) = x(n+1)$
d- $y(n) = 1/3[x(n+1)+x(n)=x(n-1)]$

Q8/ Consider the DTS

$$X(n)=\begin{cases} 1 & 0\leq n\leq 3 \\ 0 & \text{else where} \end{cases}$$

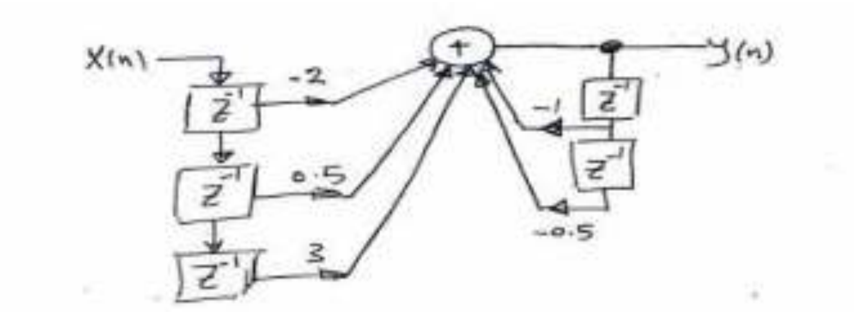
Find X(-n), x(-n-1), X(-n+1), x(n-1)+X(n+1), 2x(n) X(2n), x(n/2)

Q9/ Sketch the Block diagram for the DTS that is described by the D. E below?

① $y(n) = a_1 X(n) - a_2 X(n-2) + b_1 y(n-1) - b2y(n-2)$

② $y(n) = 3x(n) - 0.5x(n-3) - 2y(n-2) + 4y(n-4)$

Q10/ Find D.E



Q11/ Determine if the systems described by D.E are Linear or Not?

① y(n) = 2x(n) + 3x(n-1)

② y(n) = 2x(n) + 3x(n-1)+1

③ y(n) = Cos X(n)

Q12/ Determine if the systems described by the following QE ilp-o/p equations are causal or Not?

① y(n) = x(-n)

② y(n) = x(n^2)

③ y(n) = $\sum_{k=-\infty}^{n} x(k)$

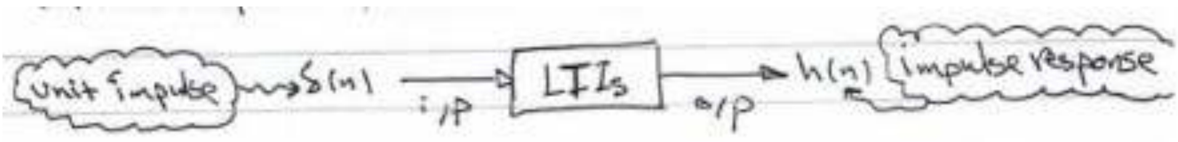
Q13/ Find and sketch the Even 4 odd Parts of the signal X(n) = Im [$e^{jn\pi/4}$]

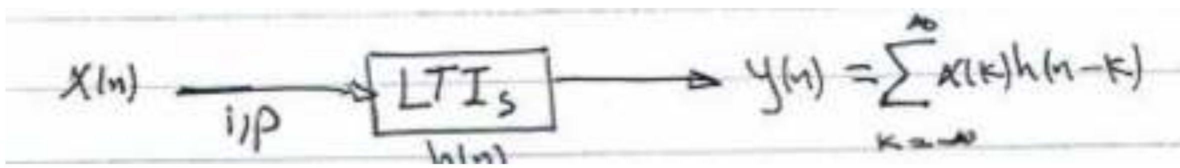
3.7 Analysis of Discrete-Time Linear Time-Invariant Systems:

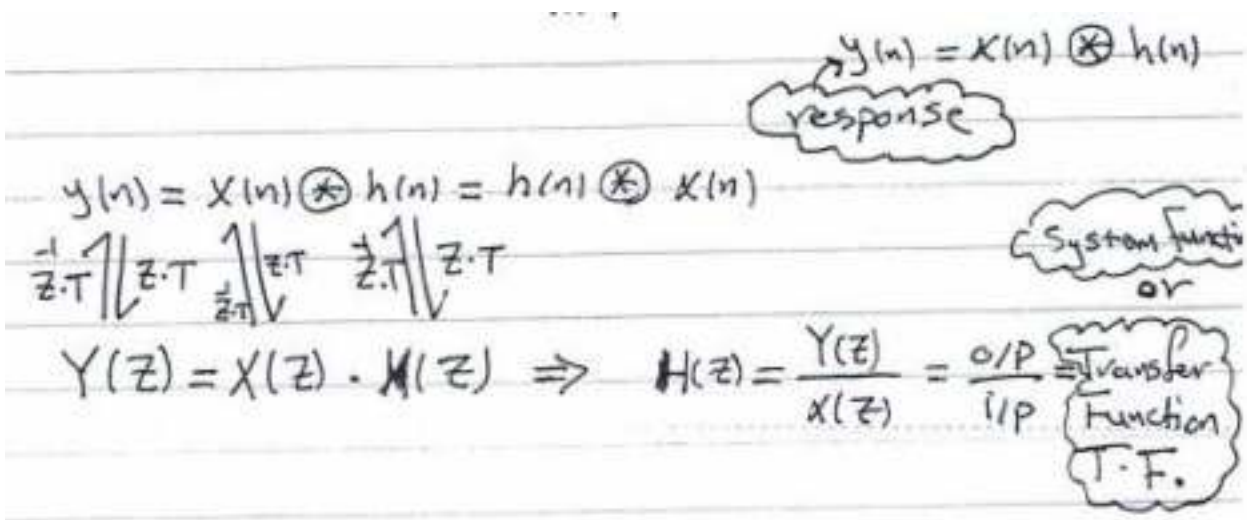
There are two basic methods for analyzing the behavior or response of a Linear time invariant System LTI to a given i/p signal.

First Method: Convolution. Sum [It is a general form of the expression that relates the Unit Sample response of the system and the arbitrary isp to o/p

Second Method: Is based on the direct solution of the input-Output equation for the System Called. [Linear Constant Coefficient Difference Equations] In this chapter we discuss the First method



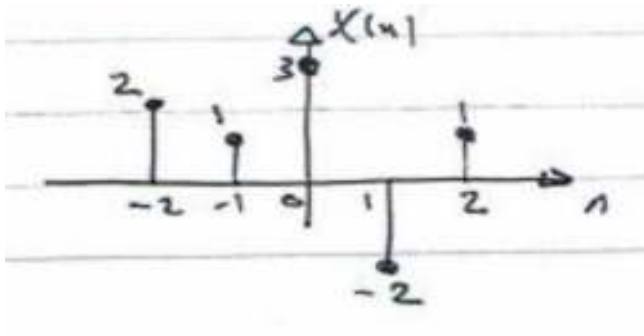




3.7.1 Discrete time Signal as weighted Impulses:

Let X(n) any arbitrary signal we can expressed in form of weightee impulses as follow

X(n)={2.1.3.-2.1}



By using : x(n)=$\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

:x(n)=$\sum_{k=-2}^{2} x(k)\delta(n-k)$

=x(-2)δ(n+2)+x(-1) $\delta(n+1)+x(0)\delta(n)+x(1)\delta(n-1)+x(2)\delta(n-2)$

X(n)=2 $\delta(n+2)+\delta(n+1)+3\delta(n)-2\delta(n-1)+\ \delta(n-2)$

Ex 1: Consider the special case of a finite - duration sequence. given as

K lnl = {2,4, 0, -2, 3} resolve the sequence Xin) into a sum of Weighted impulse Sequences.?

Sol:

X(n)=$\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)=\sum_{k=-1}^{3} x(k)\delta(n-k)$

=x(-1)$\delta(n+1)+x(0)\delta(n)+x(1)\delta(n-1)+x(2)\delta(n-2)+x(3)\delta(n-3)$

X(n)=2$\delta(n+1)+4\delta(n)-2\delta(n-2)+3\delta(n-3)$

3.7.2 The Convolution Sum:

[First Method to Analysis DT.LTISYS] The response of LTI Sys to isp Xin) into a weighted Sum of impulses Called Convolution Sum.

Y(n)=x(n)*h(n)=h(n)*x(n)

Y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n) - k = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = T(x(n))$

Computation of Convolution

①Folding: h(k) h(-k) or X (K)x(-k)

②shifting: shifting h(-k) or X(-k) by 'n'

③ Multiplication & Summation, repeat(2) until the Convolution Sum = 0.

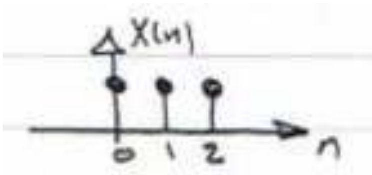
Types of Convolutions:

① Graphical Method

② Analytical Method — Linear Convolution [Non periodic seq.] / Circular Convolution [periodic seq.]

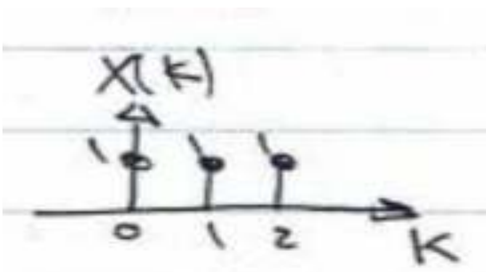
③ Table Method

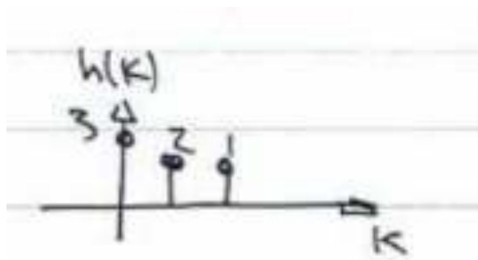
EX2: Find the Convolution Sum using Graphical Method?in S.R.4G.R



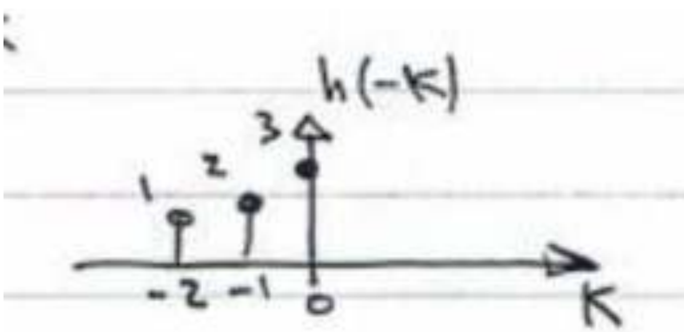
SOL:

1-

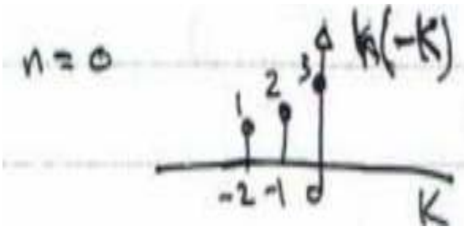


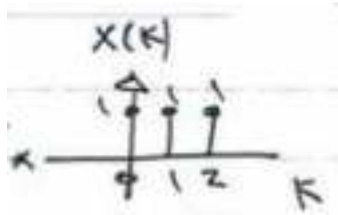


2-Folding



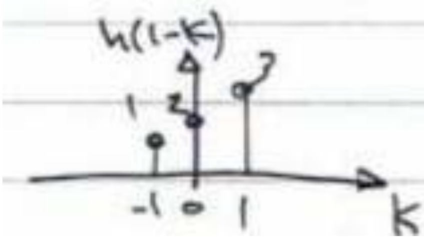
3-shifting by ”n”   N=N1+N2-1 =3+3-1=5

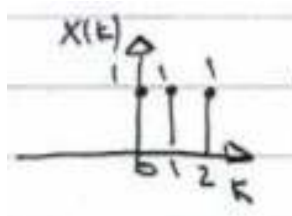




Y(0)=3*1=3

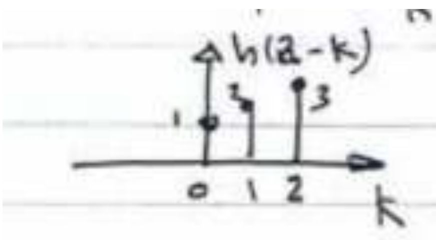
n=1

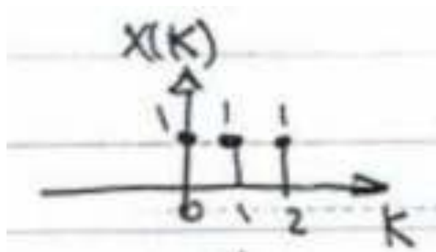




y(1)=1*2+3*1=5

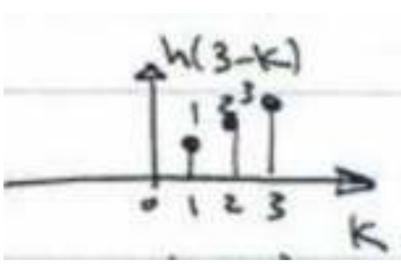
n=2

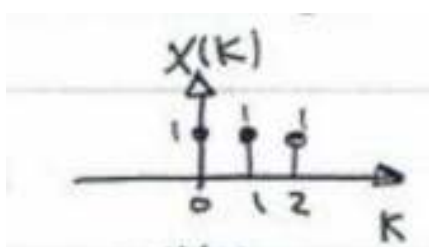




y(2)=1*1+2*1+3*1=6

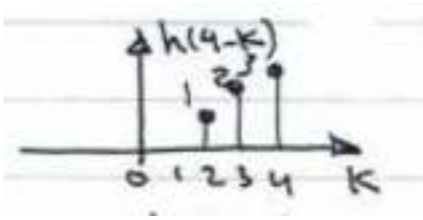
n=3

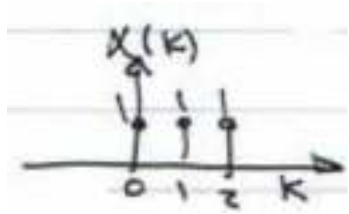




Y(3)=1*1+2*1=3

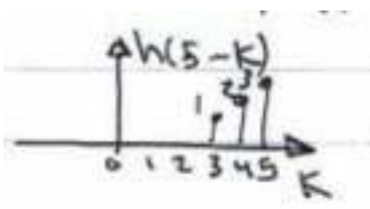
n=4

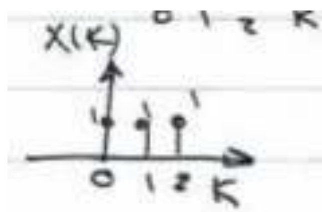




Y(4)=1*1=1

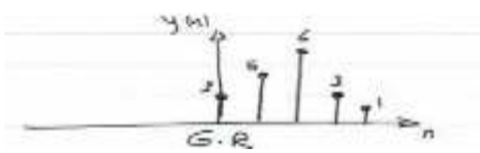
n=5





Y(5)=0

:y(n)= (3,5,6,3,1)



S.R

EX3: Determine the response for the following sequences Using Linear Convolution?

X(n)=(1,2,3,4)

h(n)=(4,3,2,1)?

Sol:

X(n) ⟶ N1=4

h(n) ⟶ N2=4 :N=N1+N2-1=7 samples

y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n-k)$

n=0   y(0)=$\sum_{k=0}^{3} x(k)h(-k) = x(0)h(0) + x(1)h(-1) + x(2)\,h(-2) + x(3)h(-3)$

=1*4+2*0+3*0+4*0=4

n=1   y(1)=$\sum_{k=0}^{3} x(k)h(1-k) = x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2)$

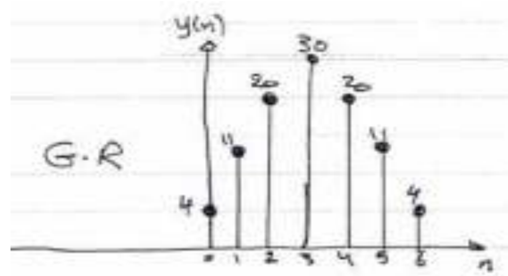
=1*3+2*4+3*0+4*0=11

n=2   y(2)=$\sum_{k=0}^{3} x(k)h(2-k) = x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1)$

=1*2+2*3+3*4+4*0=20

n=3   y(3)=30 ,n=4   y(4)=20 ,n=5 y(5)=11 ,n=6 y(6)=4

S.R y(n)=(4,11,20,30,20,11,4)



EX 4: Determine the response for the following sequences using Circular Method?

X(n)=(1,2,3,4)   h(n)=(4,3,2,1)?

Sol:

X(n) ⟶ N1=4

h(n) ⟶ N2=4 : N=N1=N2=4 samples

y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n-k)$

n=0 y(0)=$\sum_{k=0}^{3} x(k)h(-k) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3)$

=1*4+2*1+3*2+4*3=24

n=1 y(1)=$\sum_{k=0}^{3} x(k)h(1-k) = x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2)$

=1*3+2*4+3*1+4*2=22

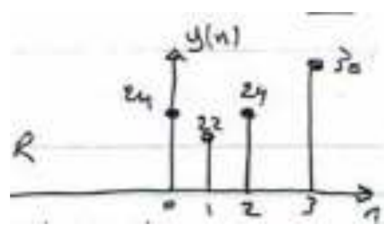
n=2 y(2)=$\sum_{k=0}^{3} x(k)h(2-k)$ =x(0)h(2)+x(1)h(1)+x(2)h(0)+x(3)h(-1)

=1*2+2*3+3*4+4*1=4

n=3 y(3)=$\sum_{k=0}^{3} x(k)h(3-k) = x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)$

=1*1+2*2+3*3+4*4=30

:S.R y(n)=[24,22,24,30]



Note: if the sequences have different lengths N1&N2

Add N2-1 zero to N1

Add N1-1 zero to N2

So that circular or linear are the same results

EX5: Find the convolution sum for the following sequences

X(n)=[1,2,3,2] h(n)=[1,1,2] in S.R4G.R

Sol:

X(n) ⟶ N1=4

h(n) ⟶ N2=3 : N=4+3-1=6 samples

N2-1=3-1=2 zero add to x(n)

N1-1=4-1=3 zero add to h(n)

:x(n)=[1,2,3,2,0,0]   N=N1=N2=6

h(n)=[1,1,2,0,0,0]

y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n-k)$

n=0 y(0)=$\sum_{k=0}^{5} x(k)h(-k) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3) + x(4)h(-4) + x(5)h(-5)$

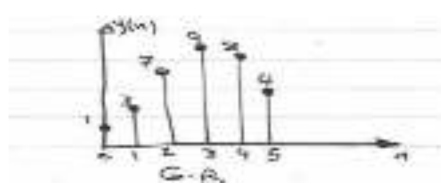
=1*1+2*0+3*0+2*0+0*2+1*0=1

n=1 y(1)=$\sum_{k=0}^{5} x(k)h(1-k) = x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2) + x(4)h(-3) + x(5)h(-4)$

=1*1+2*1+3*0+2*0+0*0+0*2=3

n=2 y(2)=7 , n=3 y(3)=9 ,n=4 y(4)=8 ,n=5 y(5)=4

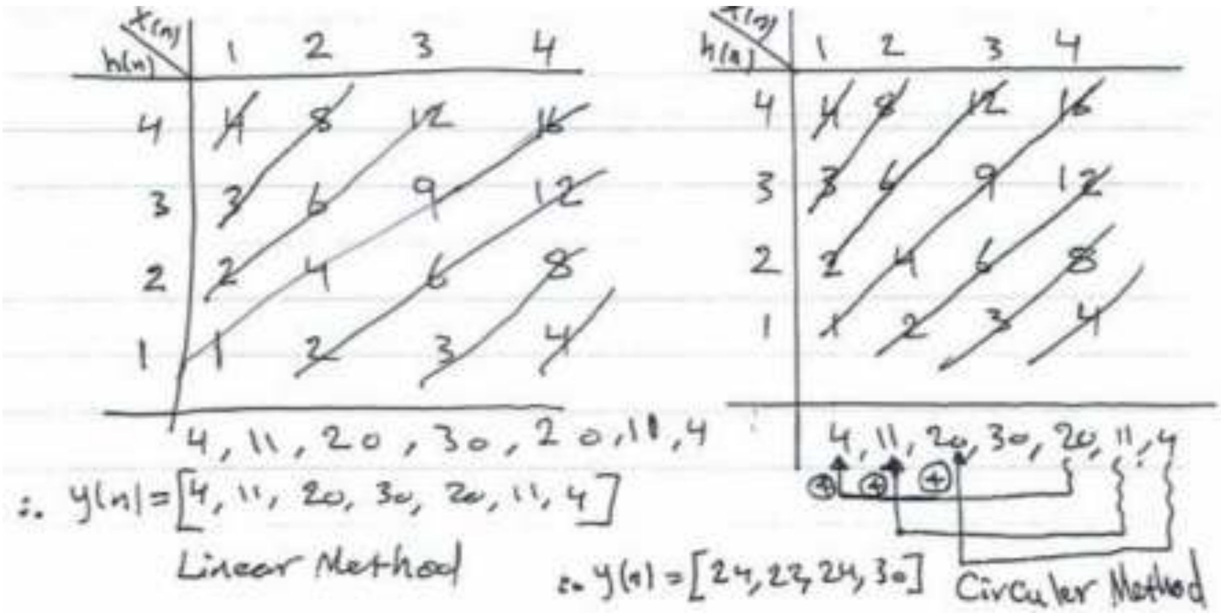
S.R y(n)=[1,3,7,9,8,4]



EX6: Find the response for following sequences?

X(n)=[1,2,3,4] h(n)=[4,3,2,1] using table method when apply linear & circular methods.

N1+N2-1=7 N1=N2=4



EX7:Find o/p by using the following sequences apply linear method ?

X(n)=[1,1,0,1,1] h(n)=[1,-2,-3,4]

n=0 (under 0 in x(n)), n=0 (under 4 in h(n))

sol: x(n) ⟶ N1=5 h(n) ⟶ N2=4 N=5+4-1=8 sample

Number of digits before zero

X(n)=2 , h(n)=3 : 2+3=5 before zero for y(n)

Y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n-k)$

n=-5

y(-5)=$\sum_{k=-2}^{2} x(k)h(-5-k) = x(-2)h(-3) + x(-1)h(-4) + x(0)h(-5) + x(1)h(-6) + x(2)h(-7)$

=1*1+1*0+0*0+1*0+1*0=1

n=-4

y(-4)=$\sum_{k=-2}^{2} x(k)h(-4-k) = x(-2)h(-2) + x(-1)h(-3) + x(0)h(-4) + x(1)h(-5) + x(2)h(-6)$

=1*-2+1*1+0+0+0=-1

n=-3 ,y(-3)=-5 , n=-2 y(-2)=2 , n=-1 y(-1)=3

n=0 y(0)=-5 ,n=1 y(1)=1 , n=2 y(2)=4

:y(n)=[1,-1,-5,2,3,-5,1,4]

properties of convolution

1-commutative property y(n)=$\sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n)$

Y(n)= $\sum_{k=-\infty}^{\infty} h(k)x(n-k) = h(n) * x(n)$

2-Associative property

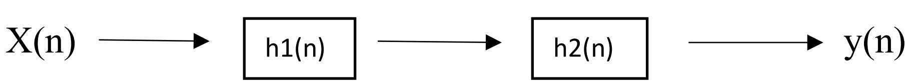
[x(n)*h1(n)]*h2(n)=x(n)*[h1(n)*h2(n)] over all unit impulse response

3-Disttibutive property

X(n)*[h1(n)*h2(n)]=x(n)*h1(n)+x(n)*h2(n)

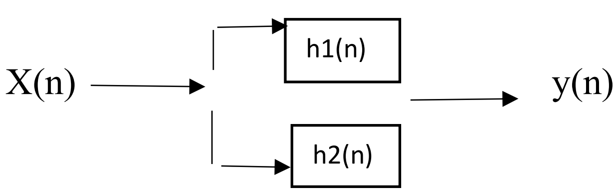
Note:

Cascade connection



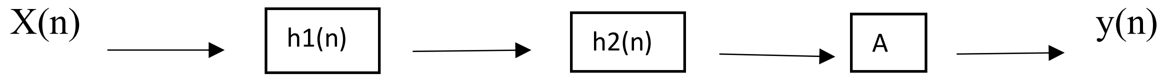
Over all unit impulse response h(n)=h1(n)*h2(n)

parallel connection



Over all unit impulse response h(n)=h1(n)+h2(n)

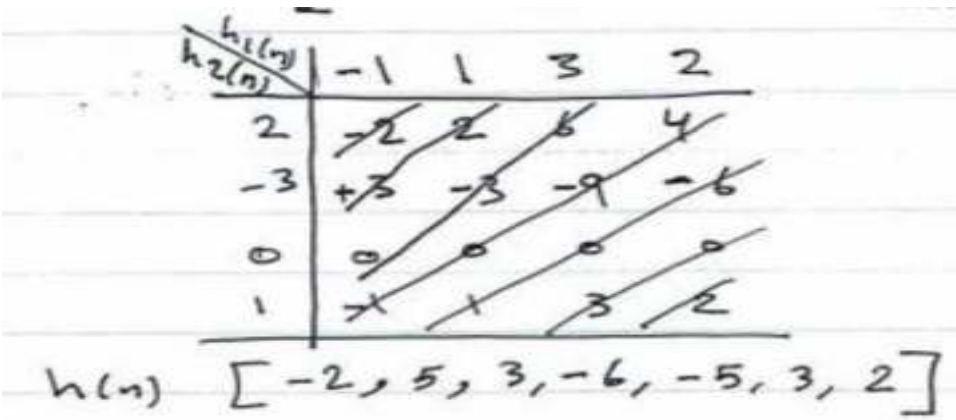
EX8: Find over all unit impulse response then find o/p response for system below:



Where $x(n)=(-2)^n\,[u(n)-u(n-3)]$

A=2 , h1(n)=[-1,1,3,2] , h2(n)=[2,-3,0,1]

Sol:



h2(n) \ h1(n)	-1	1	3	2
2	-2	2	6	4
-3	+3	-3	-9	-6
0	0	0	0	0
1	-1	1	3	2

h(n) [-2, 5, 3, -6, -5, 3, 2]

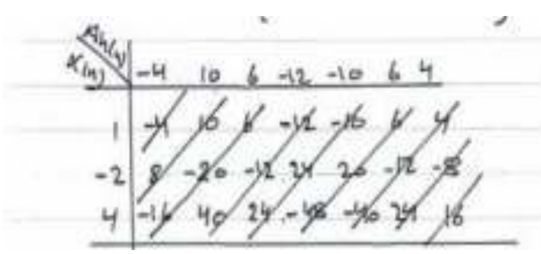
Over all impulse responses

$x(n)=(-2)^n\,[u(n)-u(n-3)]$

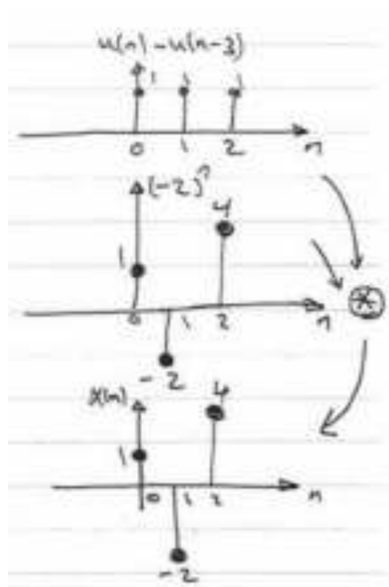
:x(n)=[1,-2,4]

:y(n)=x(n)*Ah(n)

Ah(n)=2h(n)=[-4,10,6,-12,-10,6,4]



x(n) \ Ah(n)	-4	10	6	-12	-10	6	4
1	-4	10	6	-12	-10	6	4
-2	8	-20	-12	24	20	-12	-8
4	-16	40	24	-48	-40	24	16



:y(n)=[-4,18,-30,16,38,-22,-48,16,16]

EX9: Determine the impulse response for the cascade of two LTI sys. Having impulse response

$H1(n)=(1/2)^n u(n)$ & $h2(n)=(1/4)^n u(n)$

Sol:

$h(n)=\sum_{k=-\infty}^{\infty} h1(k)h2(n-k)$

$h(n)=\sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{4}\right)^{n-k}$

$h(n)=\left(\frac{1}{4}\right)^{\wedge} n \sum_{k=0}^{n} 2^k$

$h(n)=(1/4)^n \, . \, (2^{n+1}-1)$

$h(n)=(1/4)^n . \frac{2^{n+1}-1}{2-1} = (1/4)^n \, [2^{n+1} -1]$

Note: $\sum_{k=0}^{N} a^k = \frac{a^{N+1}-1}{a-1} = \frac{1-a^{N+1}}{1-a}$

3.8 Correlation of DT sys:

The Correlation technique is very much similar to Convolution. Its provides the information about Similarity between the two Sequences. Its used in many applications where digital Signal extraction is required such radar, Communication sys Spread spectrum Comm. Mobile Comm.

Types of Correlation

a- Cross Correlation: Between two sequences X(n) Y(n) its denoted by Rxy (ℓ)

$Rxy(\ell)=\sum_{n=-\infty}^{\infty} x(n)y(n-\ell)$, ℓ=0,+1,-1, +2,-2,…….

Or

$Rxy(\ell)=\sum_{n=-\infty}^{\infty} x(n+\ell)y(n)$

Or denoted by Ryx(ℓ)

Ryx(ℓ)= $\sum_{n=-\infty}^{\infty} y(n)x(n-\ell)$

or

Ryx(ℓ)=$\sum_{n=-\infty}^{\infty} y(n+\ell)x(n)$

Note:

1-Rxy(ℓ)≠Ryx(ℓ)

Rxy(ℓ)=Ryx(-ℓ)

} correlation is not commutative

2-Rxy(ℓ)=x(ℓ)*y(ℓ)

b-Auto correlation: apply on the same sequence H is denoted by Rxx(ℓ)

Rxx(ℓ)=$\sum_{n=-\infty}^{\infty} x(n)x(n-\ell)$

Or

Rxx(ℓ)=$\sum_{n=-\infty}^{\infty} x(n+\ell)x(n)$

EX1:Deremine the cross correlation sequence Rxy(ℓ) of the following sequences x(n)=[1,-2,3,2,1]

Y(n)=[1,-1,3,2]

Sol:

-3 -2 -1 0 1

X(n)=[1 , -2 , 3 , 2 ,1] N1=5

-1 0 1 2

Y(n)=[1 , -1 , 3 , 2] N2=4 : N=N1+N2-1=8 samples

Rxy(ℓ)=$\sum_{n=-\infty}^{\infty} x(n)y(n-\ell) = \sum_{n=-3}^{1} x(n)y(n-\ell)$

Rxy(ℓ)=x(-3)y(-3-ℓ)+x(-2)y(-2-ℓ)=x(-1)y(-1-ℓ)+x(0)y(-ℓ)+x(1)y(1-ℓ)

Rxy(-5)= x(-3)y(-3+5)+x(-2)y(-2+5)=x(-1)y(-1+5)+x(0)y(5)+x(1)y(1+5)

Rxy(-5)=x(-3)y(2)+x(-2)y(3)+x(-1)y(4)+x(0)y(5)+x(1)y(6)

0 0 0 0

Rxy(-5)=1*2 =2

also

Rxy(-4)=-1 , Rxy(-3)=-1 ,Rxy(-2)=16 ,Rxy(-1)=3 ,Rxy(0)=4 ,Rxy(1)=1 ,Rxy(2)=1

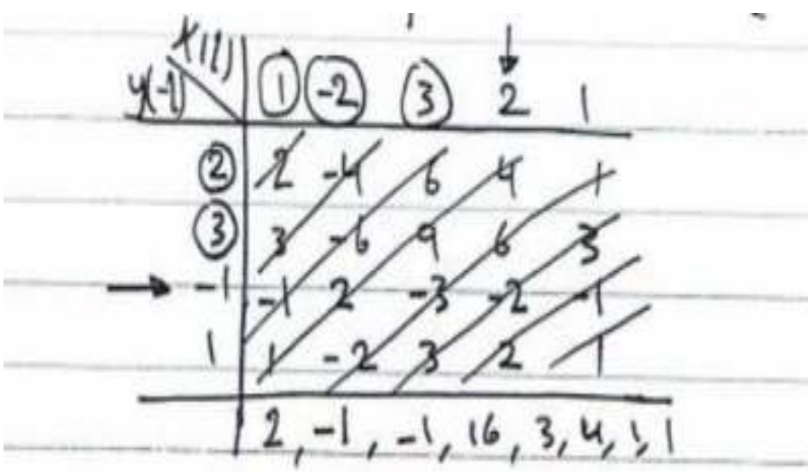
$$: Rxy(\ell) = [2, -1, -1, 16, 3, 4, 1, 1]$$

EX2:Determine the cross correlation sequence Rxy(ℓ) of the following sequences using convolution method

$$X(n) = [1, -2, 3, 2, 1] \quad y(n) = [1, -1, 3, 2]$$

$$Sol: by\ using\ note\ Rxy(\ell) = x(\ell) * y(-\ell)$$

$$X(\ell) = [1, -2, 3, 2, 1] \; y(-\ell) = [2, 3, -1, 1]$$



$$X(\ell) * y(-\ell) = [2, -1, -1, 16, 3, 4, 1] = Rxy(\ell)$$

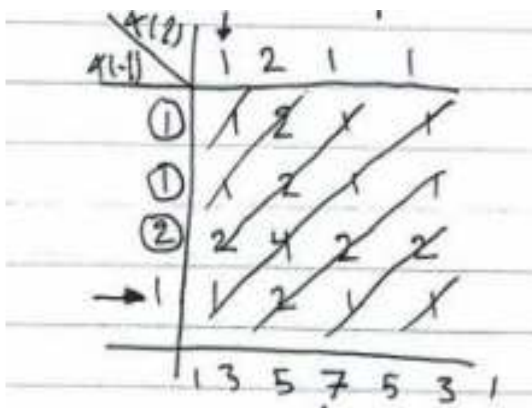
EX3:Find the auto correlation sequences for

$$X(n) = [1,2,1,1]$$

$$Sol: Rxx(\ell) = \sum_{n=-\infty}^{\infty} x(n)x(n-\ell) = x(\ell) * x(-\ell)$$

$$X(\ell) = [1,2,1,1]$$

$$X(-\ell) = [1,1,2,1]$$



$$: Rxx(\ell) = [1,3,5,7,5,3,1]$$

Sheet 4

Q1/Find the y(n) for sequences below using circular method?

$$X(n) = [1,1,0,1,1]\ , h(n) = [1,-2,-3,4]$$

Q2/ Find y(n) from the following Sequencer using easy method?

$$X(n) = [1,1,0,1,1]\ , h(n) = [1,-2,-3,4]$$

Q3/ Determine the response of the System to the i/p Signal X(n)=[1,2,3,1] which Impulse response of a Linear time invariant sys is h(n) = [1,2,1,-1] using Graphical Method

Q4/ Determine the response of the sys whose i/p x(n) and h(n) are given as follows:

$$X(n) = \begin{cases} 1/3\ n & \text{for } 0 \le n \le 6 \\ 0 & \text{else where} \end{cases}$$

$$h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{else where} \end{cases}$$

Q5/Determine the response of sys whose Unit Sample response and input are given as follows:

$$X(n) = u(n+1) - u(n-4) - \delta(n-5)$$

$$h(n) = [u(n+2) - u(n-3)].(3 - |n|)$$

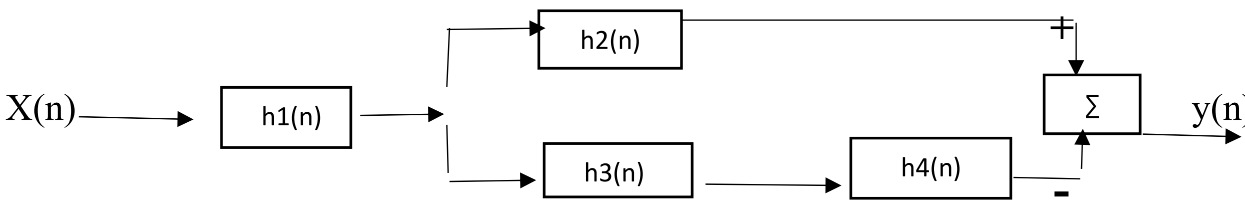
Q6/ prove that the o/p of LT I sys whose i/p and unit Sample response are given as follows:

$$X(n) = bn\, u(n)$$

$$h(n) = an\, u(n)$$

is equal to $\frac{b^{n+1}-a^{n+1}}{b-a}$ for n≥0 and a≠b

Q7/consider the inter connection of LTI sys



a)-Express the overall impulse response interns of h1(n) , h2(n) , h3(n) , h4(n)

b)-Determine h(n) when

$$h1(n) = [1/2, 1/4, 1/2]$$

$$h2(n) = h3(n) = (n+1)u(n)$$

$$h4(n) = \delta(n-2)$$

c)-Determine the response of sys if

$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-2)$$

3.9 Linear Constant Coefficient Difference Equation [Second Method to Analysis ST. LTI Sys]

In this section we will introduce DE which is an efficient Way to implement DT sys.

In Last section, the convolution Sum is:

$$Y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

If LTI sys is causal h(k)=0 for k<0

$$\therefore y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

Two types of system are possible depending upon the length of h(k)

1) Finite impulse response sys [FIR]:

Here h(k)=0 outside of some finite time interval let it be `M`

$$h(k) = 0 \quad for \;\; k \geq M$$

$$\therefore y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

$$Y(n) = h(0)x(n) + h(1)x(n-1) + \cdots \ldots + h(M-1)x(n-M+1)$$

So it is possible to implement FIR sys using number of memory location equal to (M-1)

2) Infinite impulse response sys [IIR] here h(k)≠0 for all values k>0

$$\therefore y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

$$Y(n) = h(0)x(n) + h1x(n-1+\cdots..+h(\infty)x(n-\infty)$$

So it is impossible to implement IIR sys practically. such IIR sys are implemented with the help of D.E

*Solution of D.E:

There are two method to find y(n) by linear constant coefficient difference equation .

1- Indirect method :by Z transform
2- Direct method :by homogeneous of particular solution of D.E

EX1:Find if the following system are causal or non causal then determine if sys is FIR or IIR ?

- *Y(n)=y(n-1)+x(n)*

Sol: n=0 y(0)=y(-1)+x(0)

n=1 y(1)=y(0)+x(1) — *past &present :sys is causal*

n=-1 y(-1)=y(-2)+x(-1)

$$let\ x(n) = \delta(n) \& y(n) = h(n)$$

$$h(n) = h(n-1) + \delta(n)$$

$$n = 0\ \ h(0) = h(-1) + \delta(0) = 1\ \ : sys\ is\ causal$$

$$n = 1\ \ h(1) = h(0) + \delta(1) = 1\ \ \ : y(-1) = 0 \& h(-1) = 0$$

$$n = 2\ \ h(2) = h(1) + \delta(2) = 1$$

$$and\ so\ on\ \ : TIR\ because\ h(n) \neq 0\ for\ all\ n > 0$$

- $h(n) = \begin{cases} -1 & n=1,2 \\ 1 & n=4 \\ 0 & \text{else where} \end{cases}$ by using $\delta(n)$ $the\ \Delta E\ is$

h(n)$=-\delta(n-1) - \delta(n-2) + \delta(n-4) : causal$

past past past

$$n = 0 \;\; h(0) = -\delta(-1) - \delta(-2) + \delta(-4) = 0$$

$$n = 1 \;\; h(1) = -\delta(0) - \delta(-1) + \delta(-3) = -1$$

$$n = 2 \;\; h(2) = -\delta(1) - \delta(0) + \delta(-2) = -1$$

$$n = 3 \;\; h(3) = -\delta(2) - \delta(1) + \delta(-1) = 0$$

$$n = 4 \;\; h(4) = -\delta(3) - \delta(2) + \delta(0) = 1$$

$$n = 5 \;\; h(5) = -\delta(4) - \delta(3) + \delta(1) = 0$$

n=6 $\longrightarrow$ *=0*

and so on :sys is FIR

Sheet 5

Q1/Find the following sys are causal or not and IIR or FIR

1-h(n)=4u(n+1)-4u(n-2)

2-y(n)=0.8y(n-1)=x(n)

3-h(n)=$(0.5)^n$ u(-n-1)

Deconvolution

If the impulse response and the output of a system are known, then the procedure to obtain the unknown input is referred to as deconvolution

Y(n)=h(0)x(n)+$\sum_{k=1}^{n} h(k)x(n-k)$……….1

Where n=0 , y(0)=h(0)x(0)

Hence

X(0)=$\frac{y(0)}{h(0)}$ ………….2

Rearranging (1) gives

X(n)=$\frac{y(n)-\sum_{k=1}^{n} h(k)x(n-k)}{h(0)}$ ……….3

Ex: calculate the input ,x(n),given as h(n)=[1,3,4,3,1]and y(n)=[1,4,8,10,8,4,1]

Sol:

X(0)=$\frac{y(0)}{h(0)}=\frac{1}{1}$ =1

X(1)= $\frac{y(1)-h(1)x(0)}{h(0)}=\frac{4-3*1}{1}$ =1

X(2)= $\frac{y(2)-h(1)x(1)-h(2)x(0)}{h(0)}=\frac{8-3*1-4*1}{1}$ =1

X(3)= $\frac{y(3)-h(1)x(2)-h(2)x(1)-h(3)x(0)}{h(0)}=\frac{10-3*1-4*1-3*1}{1}$ =0

And for n≥ 3 ,x(n)=0

Thus x(n)=[1,1,1]

Fourier Analysis

In Mathematics, Fourier analysis is the study of the way general Functions May be represented or approximated by sums of Simpler trigonometric Functions. Fourier analysis grew from the study of Fourier series, and is named after Joseph Fourier (1768- 1830). However, a signal in Fourier Cam be either Continuous or discrete, and It Com be either periodic or a periodic (Non-periodic). The Combination of these two features generates the four. Categories of Fourier analysis

classified as follows

1)-Fourier Series (FS)

This version of the Fourier transform used for periodic-Continuous signals that repeats itself in a regular pattern from -00 to t Ex: Sine Waves, Square Wales...etc.

2)-Fourier Transform (FT)

This Version of the Fourier transform used for Aperiodic -Continuous Signals that extend to-∞ & ∞ without repeating

Ex: de caying exponentials and the Gaussian Curre, pulse signal.

3)-Discrete Time Fourier Transform (DTFT)

This version of FT used for Aperiodic- Discrete signal that only defined at discrete points between -∞& t ∞ and do Not repeat themselves.

4)-Discrete Fourier Transform (DFT)

This version of FT used for Periodic-Discrete signal that repeat themselves from- ∞ to ∞ . Sometimes Called the Discrete Fourier Series (DFs)

Discrete Fourier Transforms DET

By using Fourier Transform FT we can find the spectrum of X(t) [analogue and Non periodic signal] which is discrete in Frequency domain

$X(F)=\sum_{-\infty}^{\infty} x(t)e^{-j2\pi ft}$ dt F.T ……..5

But we Can Not Calculating X(F) by Computer because x(t) is Continuous in time domain so we go to discrete the i/p signal by replacing with nTs . this is DTFT

$X(w)=\sum_{n=-\infty}^{\infty} x(nTs)e^{-j2\pi fnTs}$ DTFT……..6

Now i/p signal is discrete but the o/p (spectrum) X(w) is Continuous . The range of w is from $-\pi \to \pi \; or \; 0 \to 2\pi$ and again we Can Not used computer. Hence It is nelessary to Compute X(w) only at discrete values of 'w`. when FT is Calculated at

Only discrete points It is Called DFT.

$X(w)=\sum_{n=-\infty}^{\infty} x(nTs)e^{-j2\pi fnTs} = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi k}{NTs}nTs}$

$:x(k)=\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}}$ DFT …….7

IDFT : $x(n)=1/N\sum_{k=0}^{N-1} x(k)e^{\frac{j2\pi kn}{N}}$ ……..8

EX4: Evaluate the DFT of the sequence X(n)=[1,0,0,1] n≥0

Sol:

$X(k)=\sum_{n=0}^{3} x(n)e^{-\frac{j2\pi kn}{4}}$

K=0 $x(0)= \sum_{n=0}^{3} x(n)e^{-\frac{j2\pi*0*n}{4}}$ =x(0)+x(1)+x(2)+x(3) =1+0+0+1= 2

K=1 $x(1)= \sum_{n=0}^{3} x(n)e^{-\frac{j2\pi*1*n}{4}} = x(0)e^{0} + x(3)e^{-j3\pi/2} = 1 + e^{-j3\pi/2} = 1+j$

K=2 $x(2)= \sum_{n=0}^{3} x(n)e^{-\frac{j2\pi*2*n}{4}} = \sum_{n=0}^{3} x(n)e^{-j\pi n} = 1+e^{-j3\pi} = 0$

K=3 $x(3)= \sum_{n=0}^{3} x(n)e^{-\frac{j2\pi*3*n}{4}} = \sum_{n=0}^{3} x(n)e^{-j3\pi n/2} = 1+e^{-j9\pi/2} = 1-j$

EX5: what is the DFT of x(n)=$\delta(n) + 0.9\delta(n)\ for\ N = 8$?

Sol:

N=8 0→ 7 $x(0) = \delta(0) + 0.9\delta(0) = 1 + 0.9 = 1.9$

$$x(1) = \delta(1) + 0.9\delta(1) = 0 + 0.9 * 0 = 0$$

X(k)=$\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}}$=$\sum_{n=0}^{7} x(n)e^{-\frac{j2\pi kn}{8}} = 0$

X(0)=$\sum_{n=0}^{7} x(n)e^{0} = \sum_{n=0}^{7} \delta(n) + 0.9\delta(n) = \left(\delta(0) + 0.9\delta(0)\right) + \left(\delta(1) + 0.9\delta(1)\right) + \left(\delta(2) + 0.9\delta(2)\right) + \cdots .. + to\ N = 7$ =1.9

X(1)=$\sum_{n=0}^{7} x(n)e^{-\frac{j2\pi n}{8}} = (\delta(0) + 0.9\delta(0)e^{0} + \cdots \ldots to\ n = 7 = 1.9$

And soon x(2),x(3)…….x(7) :x(k)=[1.9,1.9,…..1.9] N=7

EX6: Evaluate the IDFT of the sequence

X(k)=[2,1+j,0,1,-j]

Sol;

X(n)=1/N$\sum_{k=0}^{3} x(k)e^{\frac{j2\pi kn}{N}}$

n=0 x(0)=1/4$\sum_{k=0}^{3} x(k)e^{\frac{j2\pi ko}{4}} = 1/4[x(0) + x(1) + x(2) + x(3)]$

=1/4[2+1+j+0+1-j]=1

n=1 x(1)=1/4$\sum_{k=0}^{3} x(k)e^{\frac{j2\pi ko}{4}}$=1/4$\sum_{k=0}^{3} x(k)e^{\frac{j\pi k}{2}} = 1/4[2 + (1 + j)e^{\frac{j\pi}{2}} + 0 + (1 - j)e^{\frac{j3\pi}{2}}$

=1/4[2+j-1-j-1] =0

n=2 x(2)=1/4$\sum_{k=0}^{3} x(k)e^{j\pi k} = 1/4[2 + (1 + j)e^{j\pi} + 0 + (1 - j)e^{j3\pi}]$

=1/4[2-(1+j)-(1-j)]=0

n=3 x(3)=1/4$\sum_{k=0}^{3} x(k)e^{\frac{j3}{2}\pi k} = 1/4[2 + (1 + j)e^{j3\pi/2} \; + 0 + (1 - j)e^{\frac{j3\pi}{2}*3}$

=1/4[2+(1+j)(-j)+(1-j)j]= 1

X(n)=[1,0,0,1]

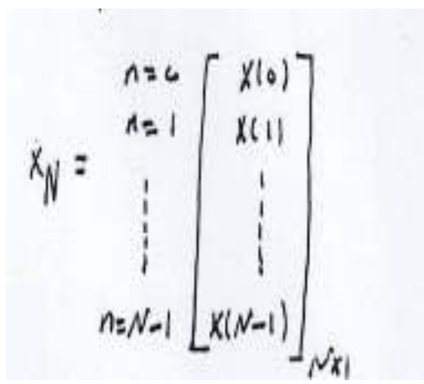
DFT as a linear transformation

Let us define ,W_N=$e^{-\frac{j2\pi}{N}}$ this is called twiddle factor. Hence DFT and IDFT equation can be written as .

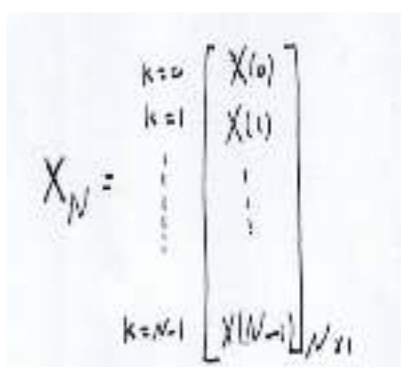
DFT: x(k)=$\sum_{n=0}^{N-1} x(n)WN^{kn}$, $k = 0,1, \dots \dots . N - 1 \dots . .9$

IDFT: x(n)=$1/N \sum_{k=0}^{N-1} x(n)WN^{-kn}$, $n = 0,1, \dots \dots . N - 1 \dots . .10$

Let us represent sequence x(n) as vector X_N of N samples i.e.,



And x(k) can be represented as a vector X_N of N samples.i.e.



The values of W_N^{kn} can be represented as matrix [W_N] of size N*N

$$W_N = \begin{bmatrix} W_N^{0\times 0} & W_N^{0\times 1} & W_N^{0\times 2} & \cdots & W_N^{0\times(N-1)} \\ W_N^{1\times 0} & W_N^{1\times 1} & W_N^{1\times 2} & \cdots & W_N^{1\times(N-1)} \\ W_N^{2\times 0} & W_N^{2\times 1} & W_N^{2\times 2} & \cdots & W_N^{2\times(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ W_N^{(N-1)\times 0} & W_N^{(N-1)\times 1} & W_N^{N-1\times 2} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}_{N\times N}$$

$k=0,1,\ldots,N-1$ (rows), $n=0,1,\ldots,N-1$ (columns)

'k' rows
'n' columns

:Then N-point DFT of equation (9) can be represented in Matrix from as .

$X_N=[W_N]x_N$

Similarly IDFT of equation (10)

$X_N=1/N\ [W_N^{k}]X_N$

Here $W_N^{kn}=[W_N]$,hence $[W_N^{k}]=W_N^{-kn}$

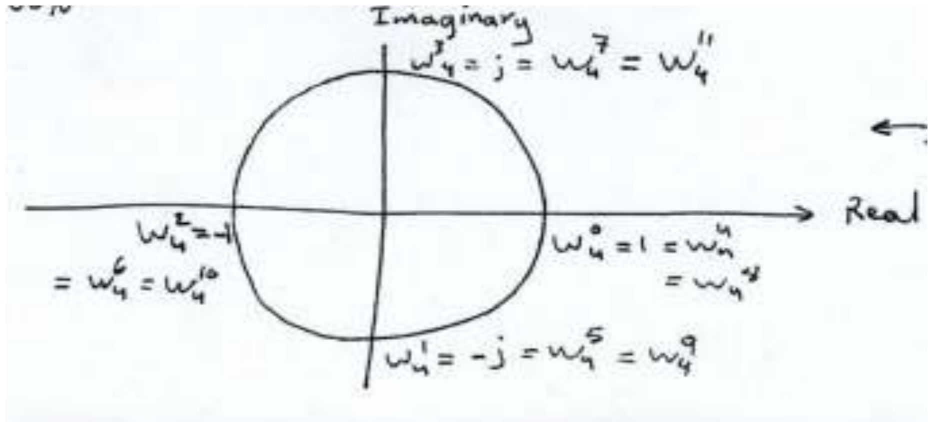
EX7: Compute DFT of the following sequence x(n)=[0,1,2,3]

Sol:

$X_N=[W_N]X_N \rightarrow x4[w4]X4$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

Now we must evaluate W_4^0 , W_4^1 , W_4^2 ,and W_4^3 from the cyclic property of W_N as follow.



$$W_N^{kn} = e^{-j2\pi kn/N}$$

$$\therefore W_4^0 = e^0 = 1 \qquad W_4^1 = e^{-j2\pi/4} = e^{-j\pi/2} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -J$$

$$W_4^2 = e^{-j2\pi 2/4} = e^{-j\pi} = \cos\pi - j\sin\pi = -1$$

$$W_4^3 = e^{-j2\pi 3/4} = e^{-j3\pi/2} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = J$$

$$\therefore W_4^0 = 1 \quad W_4^1 = -J \quad W_4^2 = -1 \quad W_4^3 = J$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -J & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -J \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

X(0)=1*0+1*1+1*2+1*3=1+2+3 =6

X(1)=1*0+ -J*1+(-1)*2+J*3=-J-2+J3 =-2+J2

X(2)=1*0+ -1*1+1*2+ -1*3=-1+2-3 =-2

X(3)=1*0+ J*1+(-1)*2+3*-J=J-2-J3 =-2-J2

:X(k)=[6,-2+J2,-2,-2-J2]

EX8: Calculate 8-point DFT of the following signal x(n)=[1,1,1,1] also calculate magnitude and phase of x(k)

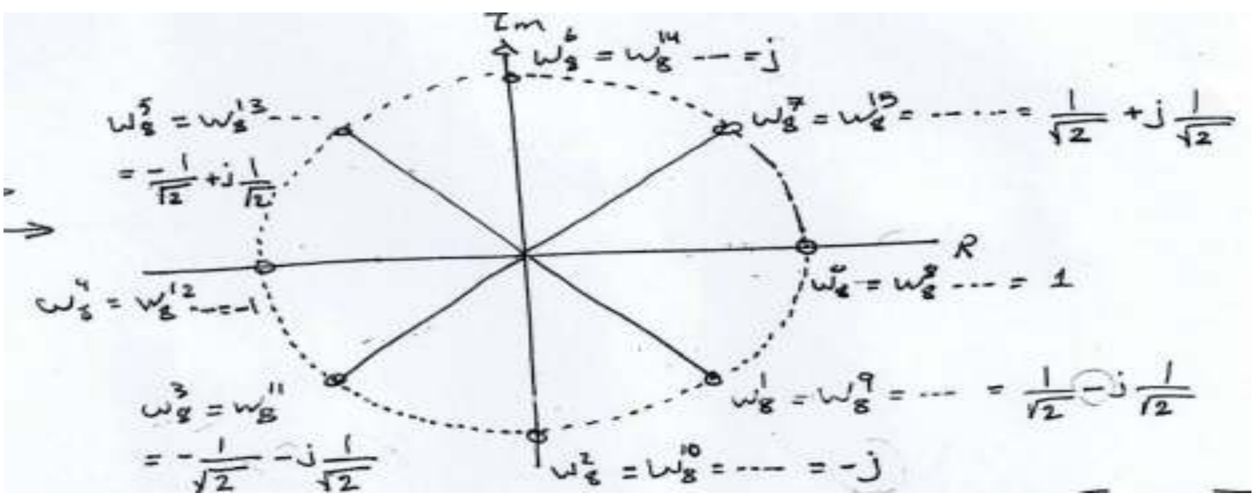
Sol:

N=8 : x(n)=[1,1,1,1,0,0,0,0]

$X_N=[W_N]X_N$

$X_8=[W_8]X_8$

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5)\\ X(6)\\ X(7)\end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0\\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7\\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14}\\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21}\\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28}\\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35}\\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42}\\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49}\end{bmatrix} \begin{bmatrix} 1\\ 1\\ 1\\ 1\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$



$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & j & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$X(0)=1+1+1+1+0+0+0+0=4$

$X(1)=1+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}-j-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}+0+0+0+0=1-j(1+\sqrt{2})$

$X(2)=1-j-1+j+0+0+0+0=0=$

$X(3)=1-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}+j+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}+0+0+0+0=1-j(\sqrt{2}+j)$

$=1+j(1-\sqrt{2}) = 1-j0.414$

$X(4)=1-1+1-1+0+0+0+0=0$

$X(5)=1-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-j+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}+0+0+0+0=1+j\sqrt{2}-j$

$$=1-j(1-\sqrt{2})$$

$X(6)=1+j-1-j+0+0+0+0=0$

$X(7)= 1+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}+j-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}+0+0+0+0=1+j\left(1+\sqrt{2}\right)$

$$=1+j2.414$$

To find magnitude and phase

"magnitude" $|x(k)| = \sqrt{[XR(k)]^2+[XI(k)]^2}$

"phase" x(k)=tan^{-1} $\frac{x1(k)}{XR(k)}$

Magnitude **Phase**

$\|x(0)\| = 4$	< X(0)=0
$\|x(1)\| = 2.613$	< X(1)=-1.178
$\|x(2)\| = 0$	< X(2)=Not calculate
$\|x(3)\| = 1.082$	< X(3)=-0.392
$\|x(4)\| = 0$	< X(4)= Not calculate
$\|x(5)\| = 1.082$	< X(5)=0.392
$\|x(6)\| = 0$	< X(6)= Not calculate
$\|x(7)\| = 2.613$	< X(7)=1.178

Properties of DFT

1-poriodicity: if X(n+N)=x(n) ,X(k+N)=x(k)

2-Linearity : if X1(n) $\xrightarrow[\boxed{N}]{DFT}$ $X1(k)$

And X2(n) $\xrightarrow[\boxed{N}]{DFT}$ X2(k)

a1x1(n)+a2x2(n)$\overset{DFT}{\underset{\boxed{N}}{\longleftrightarrow}}$a1x1(k)+a2x2(k)

3-Symmetry for read valued x(n):

If x(n) is read, then

X(N-K)=X(k)=X(-k)

4-Circular convolution :

If x1(n) $\overset{DFT}{\underset{\boxed{N}}{\longleftrightarrow}}$ x1(k) and x2(n) $\overset{DFT}{\underset{\boxed{N}}{\longleftrightarrow}}$ x2(k)

Then $x1(n) \boxed{N} x2(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x1(k)x2(k)$

5-Time Reversal of sequence :

If $x(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x(k)$ then $x(N\text{-}n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x(N\text{-}k)$

6-Delay [Time shift]

If $x(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x(k)$ then $x(n\text{-}n_o) \overset{DFT}{\longleftrightarrow} x(k)e^{-j2\pi kno/N}$

7-Circular correlation :

If $x(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x(k)$ and $y(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} y(k)$ then $x(n)*y(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x(k)\ y(k)$

8-Muliplication of two sequences:

If $x1(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x1(k)$, $X2(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} x2(k)$

Then $x1(n).x2(n) \underset{\boxed{N}}{\overset{DFT}{\longleftrightarrow}} 1/N\ x1(k) \boxed{N} x2(k)$

EX9: The first five points of the 8-point DFT of a real valued sequence are (0.25,-j0.3018 ,0, 0, 0.125-j0.0518) Determine remaining three points of the DFT.

Sol:

X(0)=0.25

X(1)=-j0.3018

X(2)=0

X(3)=0

X(4)=0.125-j0.0518

So the remaining points i.e x(5),x(6),x(7) are to be determined. The time domain sequence is real valued .the symmetry property for the real valued x(n)is given by

X(N-k)=x*(k)

:N=8 , :x(8-k)=x*(k)

By take complex conjugates of both sides

X*(8-k)=x(k)

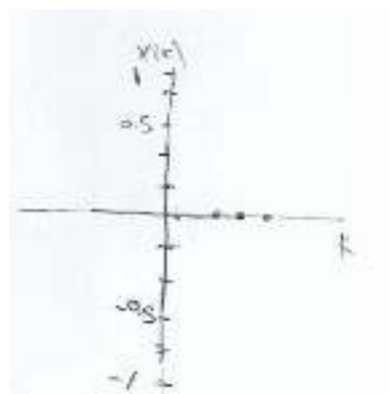
X(k)=x*(8-k)

For k=5 ,x(5)=x*(8-5)=x*(3)=0

For k=6 ,x(6)=x*(8-6)=x*(2)=0

For k=7 ,x(7)=x*(8-7)=x*(1)=j0.3018

:x(k)=[0.25,-j0.3018,0 ,0 ,0.125-j0.0518 ,0, 0,j0.3018]



EX10: Compute the circular convolution of x1(n)=[2,1,2,1] & x2(n)=[1,2,3,4]

Using DFT and IDFT?

Sol:

To calculate circular convolution of x1(n) & x2(n)

1- $x1(n) \xleftrightarrow[N=4]{DFT} x1(k)$

2- $x2(n) \xleftrightarrow[N=4]{DFT} x2(k)$

3-x3(k)=x1(k) x2(k)

4-x3(n)=IDFT[x3(k)] , x3(n):is the circular convolution of x1(n)&x2(n)

(i)-to compute DFT of x1(n)

X1(n)=[2,1,2,1]

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

x1(0)=2+1+2+1=6

x1(1)=2-j1-2+j1=0

x1(2)=2-1+2-1=2

x1(3)=2+j-2-j=0

(ii)-to compute DFT of x2(n)

X2(n)=[1,2,3,4]

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

X2(0)=1+2+3+4=10

X2(1)=1-j2-3+j4=-2+j2

X2(2)=1-2+3-4=-2

X2(3)=1+j2-3-j4=-2-j2

(iii)-Multiply two DFTs x1(k) & x2(k):

X3(0)=6*10=60

X3(1)=0*(-2+j2)=0

X3(2)=2*-2=-4

X3(3)=0*(-2-j2)=0

:x3(k)=[60,0,-4,0]

(iv)-To obtain x3(n)by IDFT of x3(k)

$X_N = 1/N[W^*_N]X_N$, $[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$X_4 = 1/4[W^*_4]X_4$,$[W_4] = [W_4^{-1}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

X3(0)=1/4[60+0-4+0]=14

X3(1)=1/4[60+0+4+0]=16

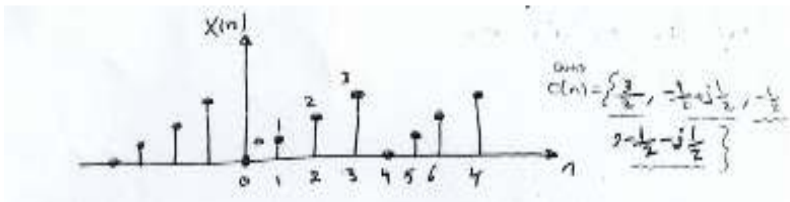
X3(2)=1/4[60+0-4+0]=14

X3(3)=1/4[60+0+4+0]=16

:x3(n)=[14,16,14,16]

Sheet6

Q1/ Find the DFS Coefficients of the signal shown below



Q2/ Find the periodic sequences X(n) From the DFS Coefficient Sequence

X(K) =[0,-j1/2 ,0, j1/2]

Ans=[0,1,0,-1]

Q3/ Find DFT of the sequence x(n)=[0,1,0,1,1] $n \geq 0$

Q4/ By means of the DFT and IDFT, determine the sequence X3(K) Corresponding to the circular convolution of the sequences X(n) and X2(n)

X1(n)=[2,1,1,1] , x2=[1,1,3,2]

Q5/ Determine the sequence y(n) for Convolution of the sequences

X1(K) & X2(K) , x1(k)=[1,4,1] , x2(k)=[2,2,2]

Q6/ Evaluate the IDFT of the sequence x(K)=[a1, a2, a3, a4]

Q7/ Given a sequence X(n) for $0 \leq n \leq 3$, where X(0) = 1, X(1) = 2,X(2)=3 and X(3) =4 Evaluate Its DFT X(K).

Fast Fourier Transforms FFT

* Computation of DFT using FFT Algorithms

We studied DFT earlier. The DFT is used in large number of applications of Dsp such as Filtering, Correlation analysis spectrum analysis etc. But the direct Computation of DFT involves large number of Computations. Hence the processor remain busy. special algorithms have been developed to compute OFT quickly .These algorithms exploit the periodicity and symmetry properties of twiddle factors (phase factors). Hence DFT is computed fast using FFT Fast Fourier Transform a algorithms.

i/p x(n)→o/p x(k) , x(n)consisting of 2^m :m=positive integer

:x(n) is power of 2

N=2,4,8,16,etc

Note: if x(n) does not Contain Samples, then we simply append It with Zeros until the number of the appended sequence is a power of 2

$X(k)=\sum_{n=0}^{N-1} x(n)\ Wn^{kn}$,k=0,1,….N-1

$X(k)=x(0)*W^0_N+x(1)*W^k_N+x(2)*W^{2k}_N+x(3)*W^{3k}_N+\ldots x(N-1)*W^{(N-1)k}_N$

:. No. of Complex Multiplication for X(K) = N*N= N^2

" "" additions " X(K) = (N-1) *N=N^2-N if N=l024 point DFT

: Complex Multiplications = N^2= $(1024)^2 \cong 1 * 10^6$

: Complex additions = N^2-N= $(1024)^2$ -1024 $\cong 1 * 10^6$

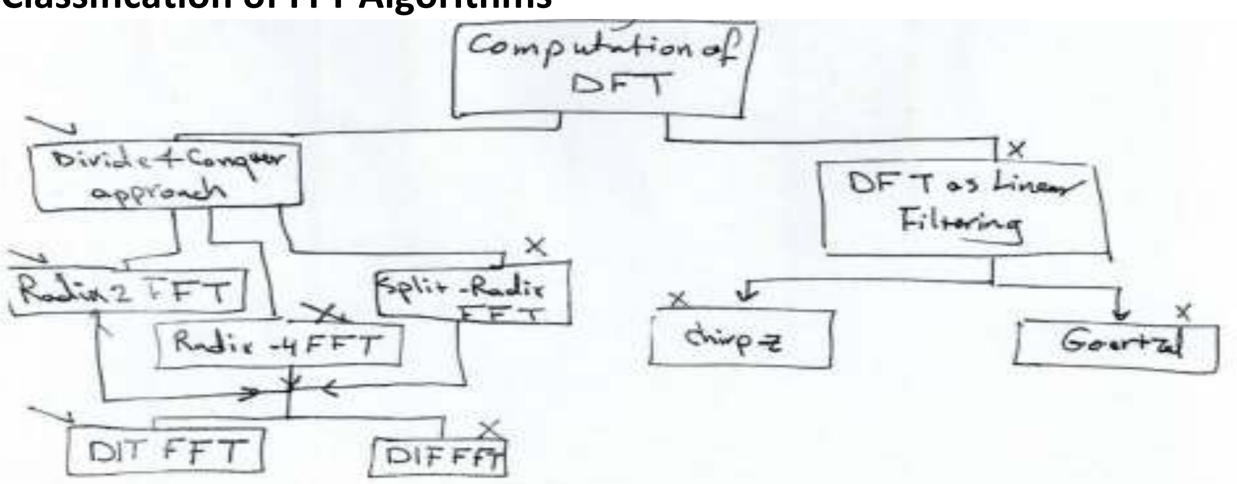
Now Let us assume that the processor executes one Complex. Multipication in 1 Microsecond.

:. Time = (Complex Multiplication *Time for one multiplication) + (Complex addition * Time for one addition) = $(1 * 10^6 * 1 * 10^{-6}) + (1 * 10^6 * 1 * 10^{-6})$

= 1+1 = 2 Seconds

Thus two Seconds of the time is required for Computations of 1024 paint DFT. In terms of processors, this is Large time. This is because processors has to do Lot of other work Such as fetching and storing data in the memory, handling data inputs and outputs, displays etc. Hence real time Computation of DFT for Large values of N becomes practically impossible by direct Computation: So we used FIT to Compute DFT.

Classification of FFT Algorithms



Radix-2 FFT Algorithms

The radix-2 FET algorithms are based on divide and conquer approach. In this approach the N- Point DFT is successively decomposed into Smaller DFTs. Because of this decomposition, the Number of Computations are reduced.

Let $N = 2^v$ where v = No. of stages of decimation

: Smallest DFT → N=2

So this type Called Radix-2

if $N=2^v$

$:v=\frac{log10^N}{log10^2}=\log_2 N$

if N=4 : $V= \log_2{}^4$ =2stage

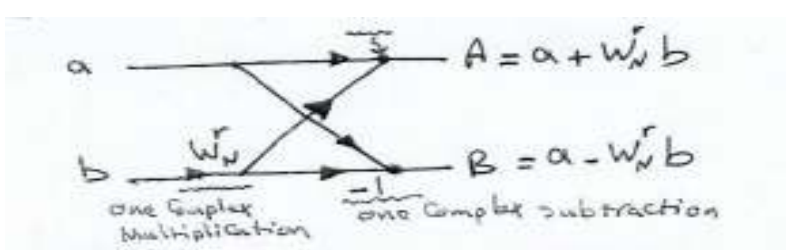
N=8 : $V=\log_2{}^8$ = 3stage

Radix=2 DIT-FFT Algorithm

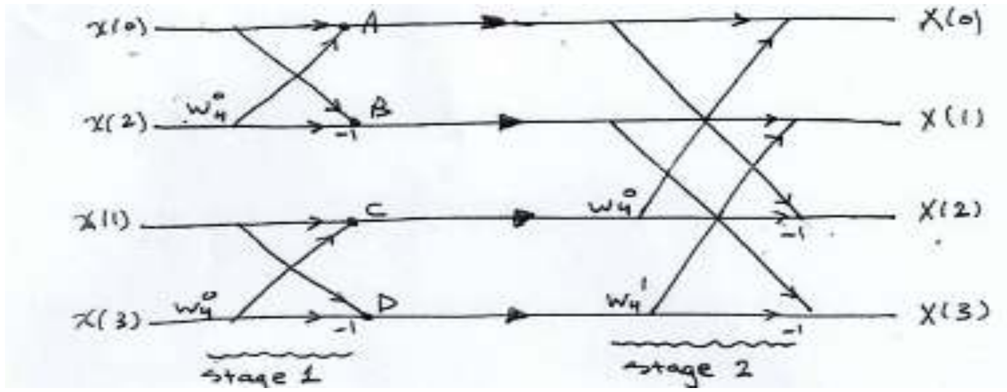
Here DIT means Decimation in Time. the time domain Sequence x(n) is Splitted into two sequences. This splitting operation is called decimation. Since It is done on time domain sequence it is Called Decimation in Time (DIT).

Butterfly Computation

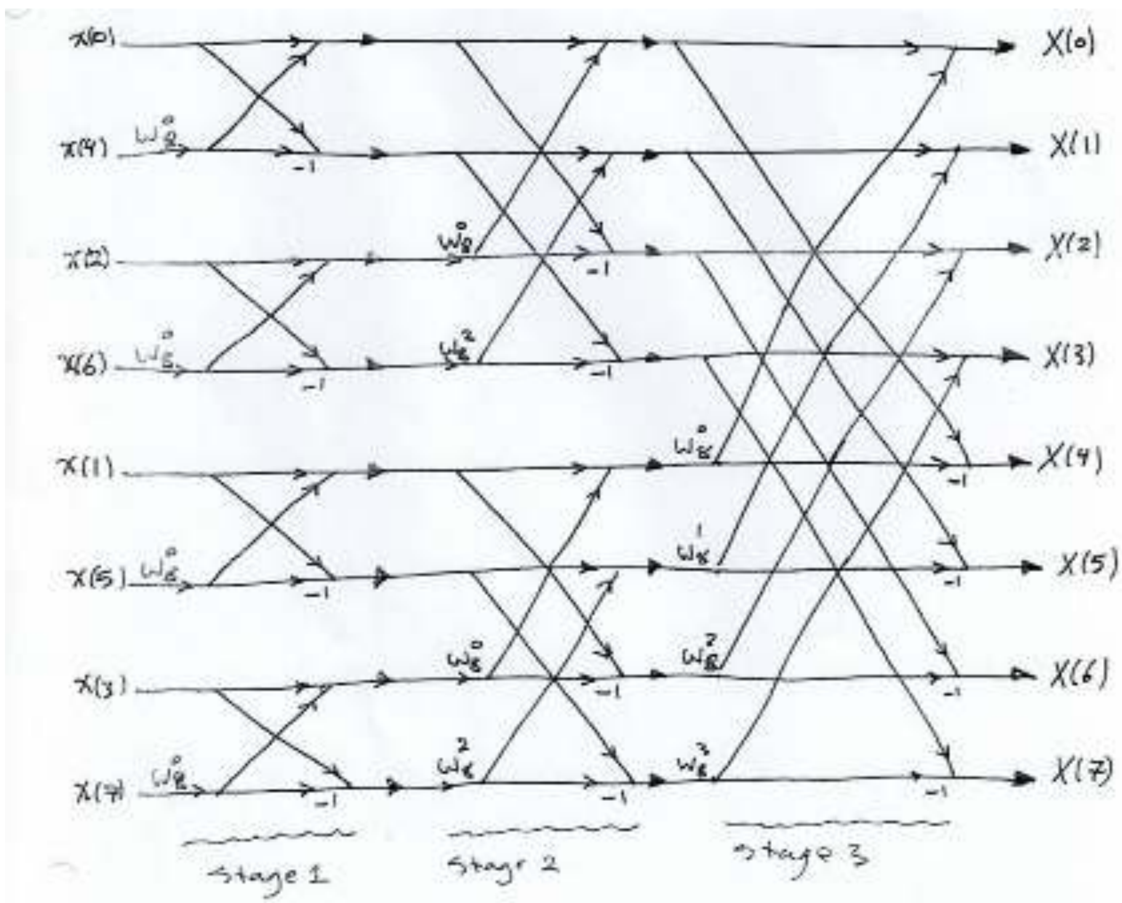
This is the fundamental or basic computation in FFT algorithms as follow:



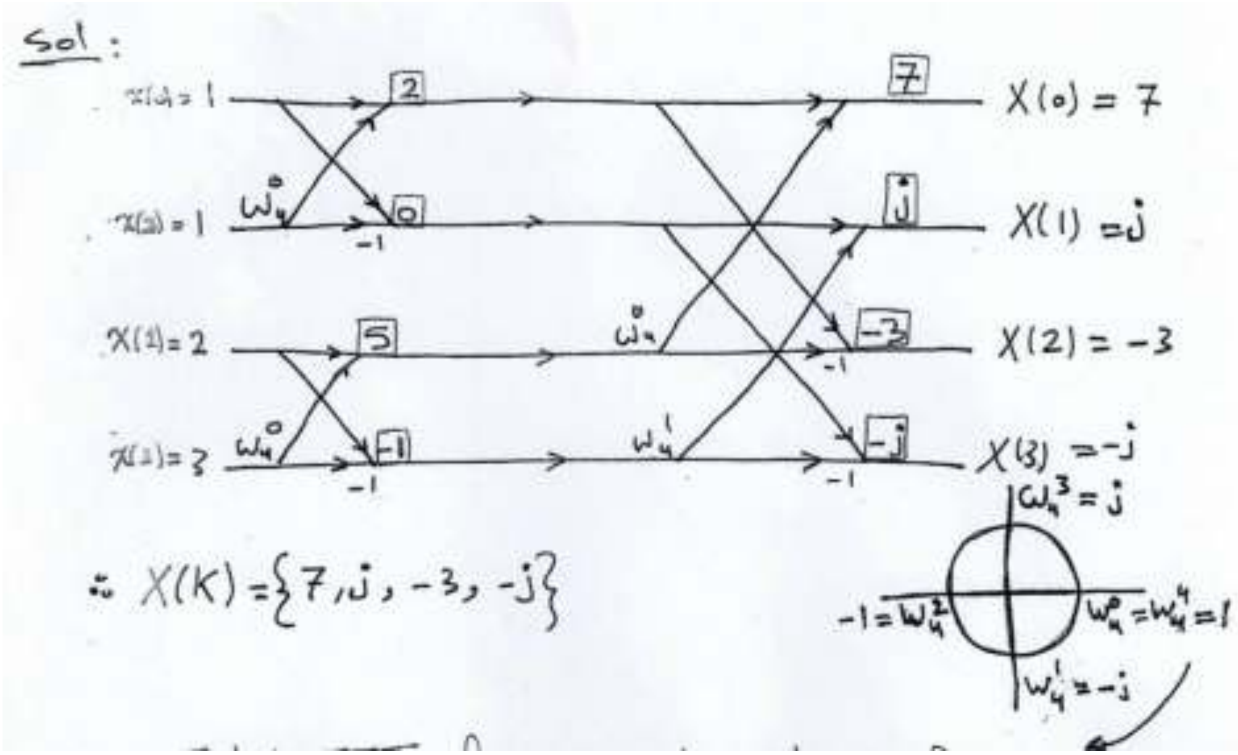
DIT-FFT for 4 point using signal flow graph



DIT-FFT for 8 point using signal flow graph

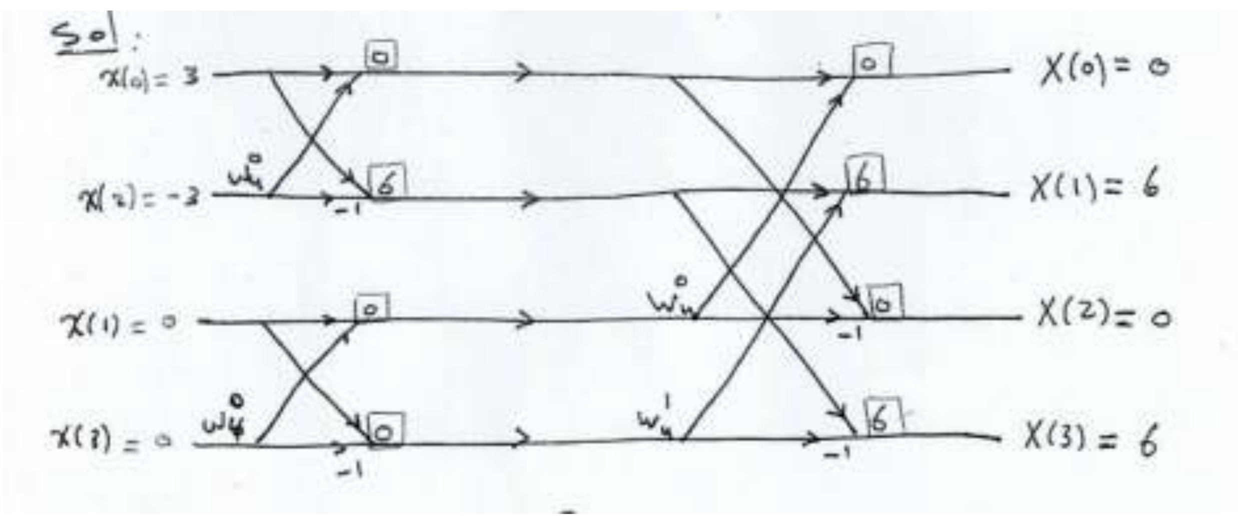


EX1: Determine FFT of x(n)=[1,2,1,3] using flow-graph with N=4 ?



EX2: Find the FFT of x(n)=3cos(0.5nπ) using flow graph with N=4?

X(0)=3 ,x(1)=0 ,x(2)=-3 , x(3)=0



: x(k)=[0,6,0,6]

EX3: Use the 8-point radix-2DIT-FFT flow graph (algorithm) to find the DFT of x(n)=[0.707 , 1, 0.707, 0, -0.707 , -1, -0.707,0]

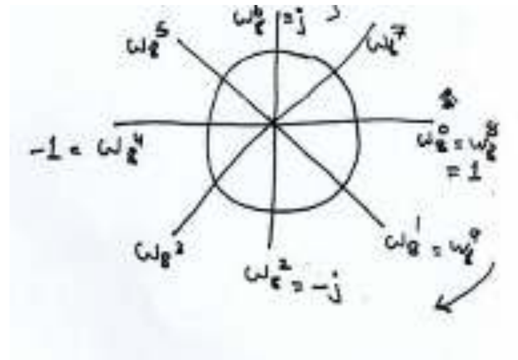
Sol:

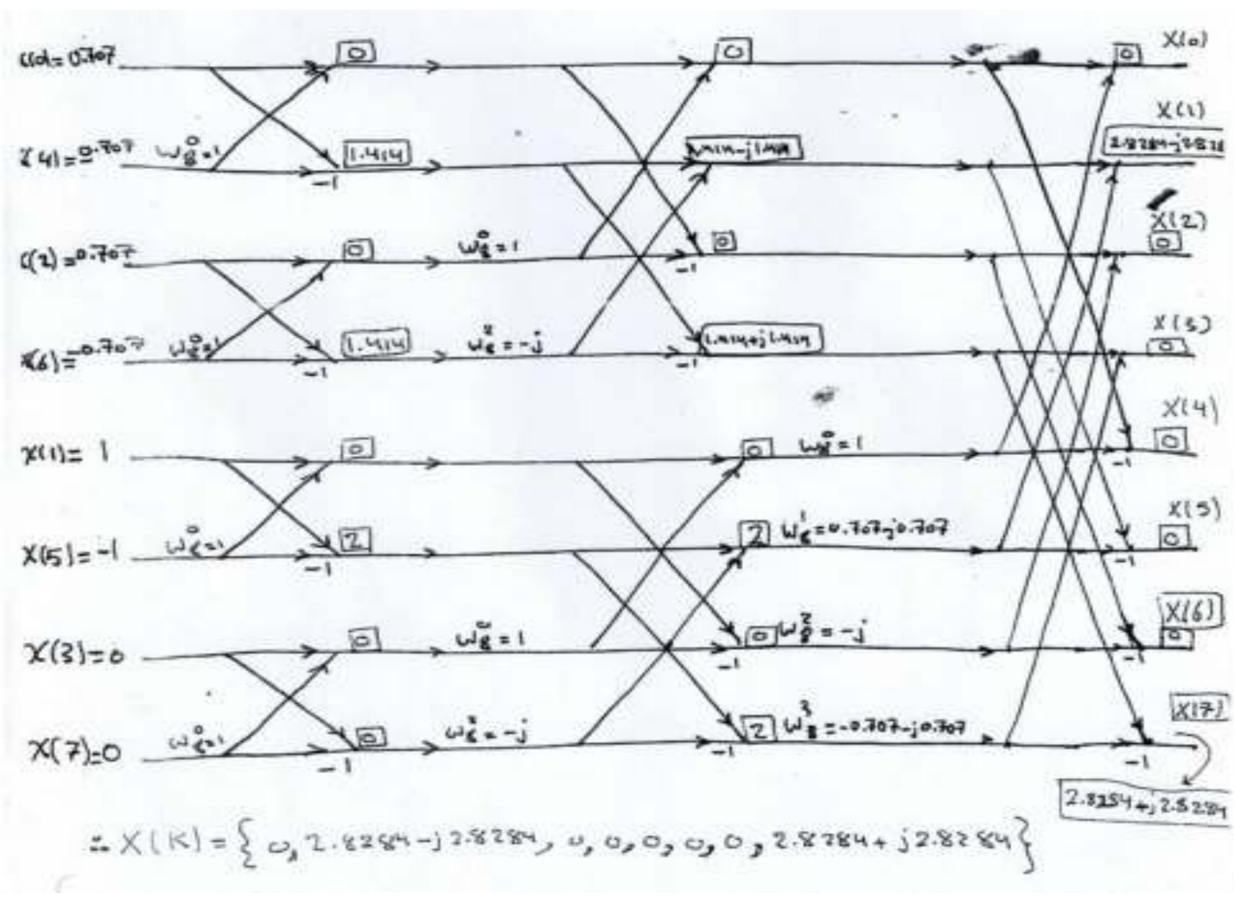
$W_8^1=W_8^9=0.707-j0.707$

$W_8^3=W_8^{11}=-0.707-j0.707$

$W_8^5=W_8^{13}=-0.707+j0.707$

$W_8^7=W_8^{15}=0.707+j0.707$





Sheet7

Q1/For m 8-point DIT-FFT algorithm. Draw the signal flow graph determine the DFT of the following sequence. X(n) =[1,1,1,1,0,0,0,0]

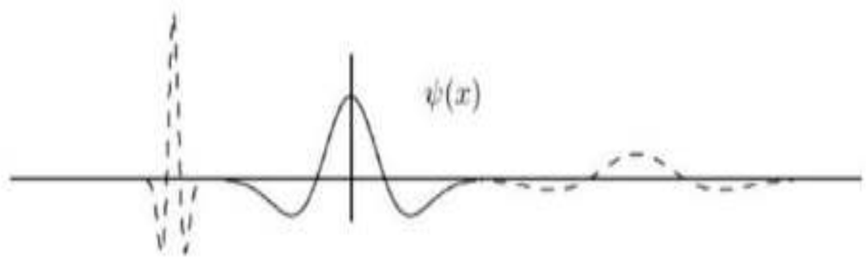
Q2/ obtain the 8-point DFT of the following sequence using Radix-2 DITT-FFT algorithm .show all the results along signal flow graph. x(n) =[2,1,2,1]

Q3/ if x(n) =[1, 2, 3,4] find X(K) using Rodix-2 DIT -FFT flow graph.

The continuous wavelet transform

- Wavelet $\varphi a, b(t) = \frac{1}{\sqrt{b}} \varphi(\frac{t-a}{b})$

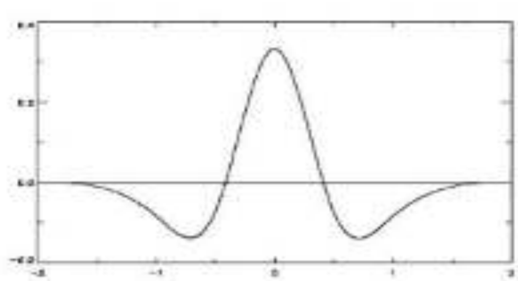
- Decomposition $W(a, b) = k \int_{-\infty}^{+\infty} \varphi * (\frac{x-b}{a}) f(x) dx$



The continuous wavelet transform

Example: the Mexican hat wavelet

$$g(x) = (1 - x^2) e^{-1/2\, x^2}$$



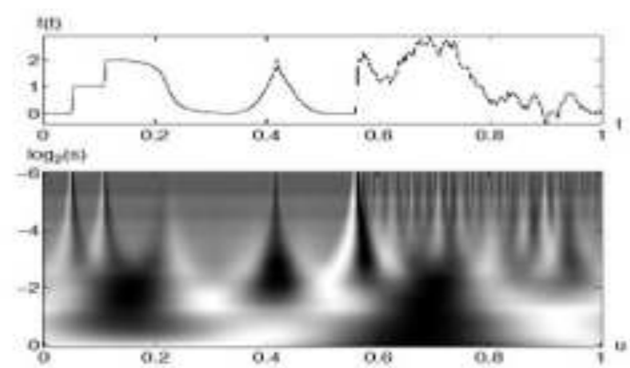


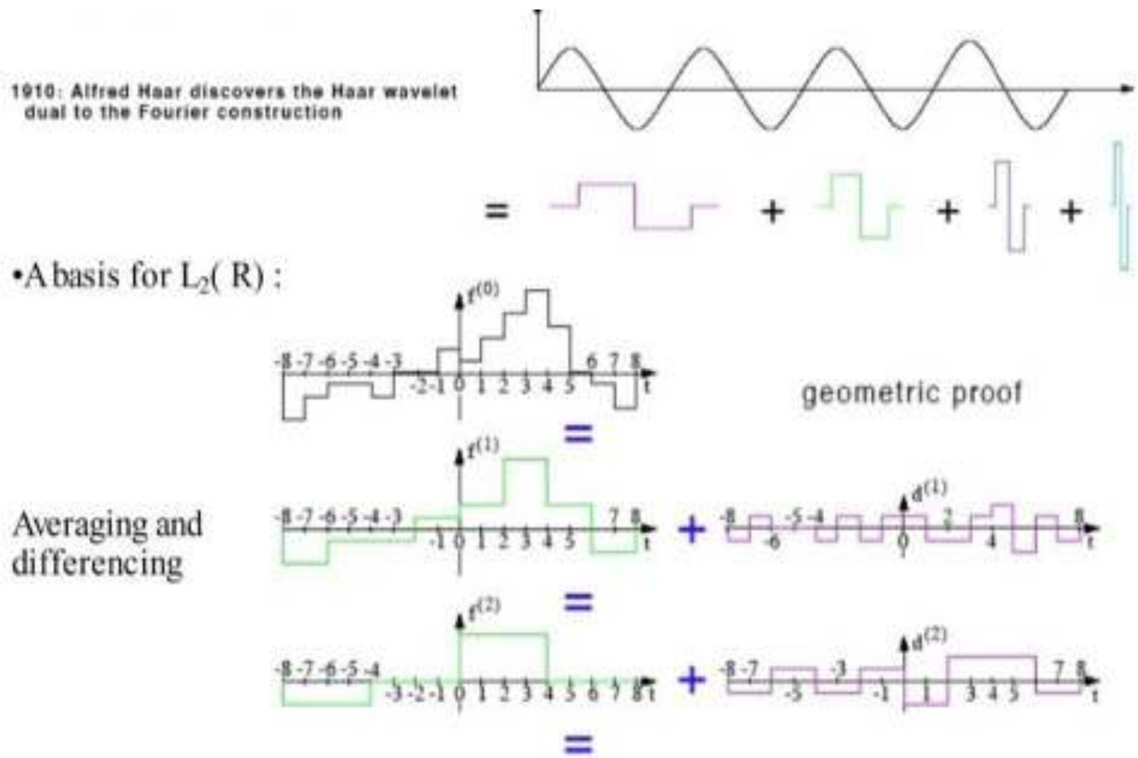
Figure 4.7: Real wavelet transform $Wf(u, s)$ computed with a Mexican hat wavelet. The vertical axis represents $\log_2 s$. Black, grey and white points correspond respectively to positive, zero and negative wavelet coefficients.

The continuous wavelet transform

- Reconstruction $f(x) = \frac{1}{c\varphi}\int_{-\infty}^{+\infty}\int_{0}^{+\infty} 1/\sqrt{a}\ w(a,b)\varphi(\frac{x-b}{a})\frac{dadb}{a^2}$
- Admissible wavelet : $c\varphi = \int_{-\infty}^{+\infty}|\Psi(t)|^2\frac{dt}{t} < +\infty$
- Simpler condition : zero mean wavelet $\varphi(0) = 0$

Practically speaking, the reconstruction formula is of no use Need for discrete wavelet transforms which preserve exact reconstruction.

The Haar Wavelet

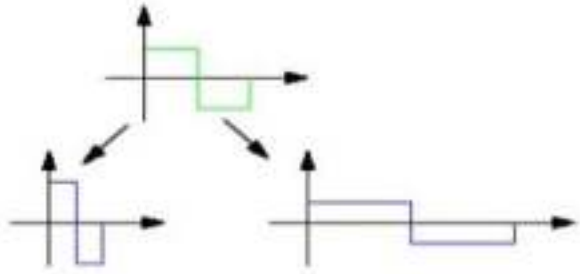


The Haar Wavelet

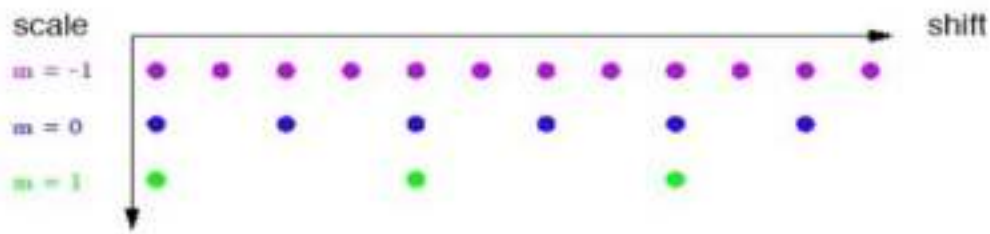
Basis functions

$$\psi(t) = \begin{cases} 1 & 0 \le t < 0.5 \\ -1 & 0.5 \le t < 1 \\ 0 & \text{else} \end{cases}$$

$$\psi_{m,n}(t) = 2^{-m/2}\psi(2^{-m}t - n)$$



Compute WT on a discrete grid



The Haar multiresolution analysis:

- A sequence of embedded approximation subsets of $I_2(R)$

$$\{0\} \leftarrow \ldots V_{j-1} \subset V_j \subset V_{j+1} \ldots \rightarrow \mathbf{L}^2(\mathbf{R})$$

with : $f(x) \in V_j$ iff $f(2x) \in V_{j+1}$

$f(t) \in V_0 \Leftrightarrow f(t-k) \in V_0,\ k \in \mathbb{Z}$

$\{\varphi(t-k)\}_{k \in \mathbb{Z}}$ forms an orthonormal basis for V_0

- And a sequence of orthogonal complements, details subspaces:

W_n such that $V_{n+1}=V_n+W_n$

- φ *is the scaling function. it's a low pass filter*

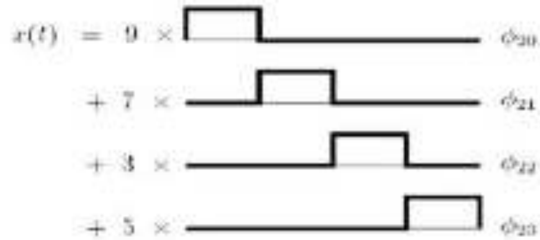
- A basis in i$V_j$ven by: $\varphi j, k(t) = 2^{\frac{j}{2}}\varphi(2^j t - k), k \epsilon z$

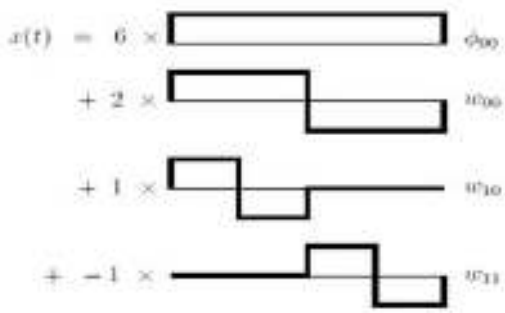
The Haar multiresolution analysis

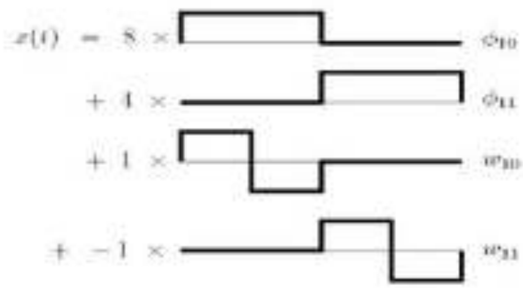
analysis

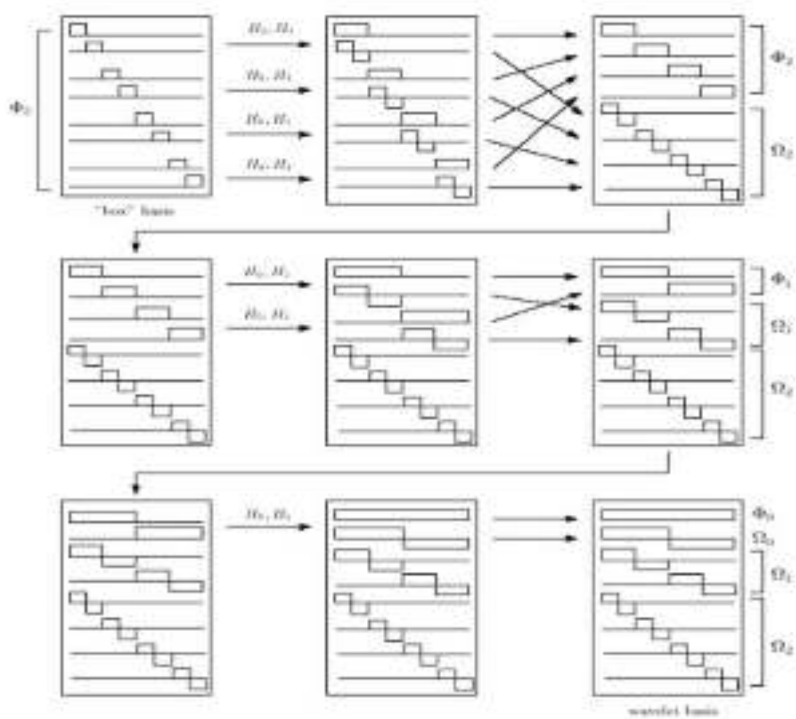
Example :

$$x(t) = \begin{cases} 9 & \text{if } t \in [0, 1/4) \\ 7 & \text{if } t \in [1/4, 1/2) \\ 3 & \text{if } t \in [1/2, 3/4) \\ 5 & \text{if } t \in [3/4, 1) \end{cases}$$

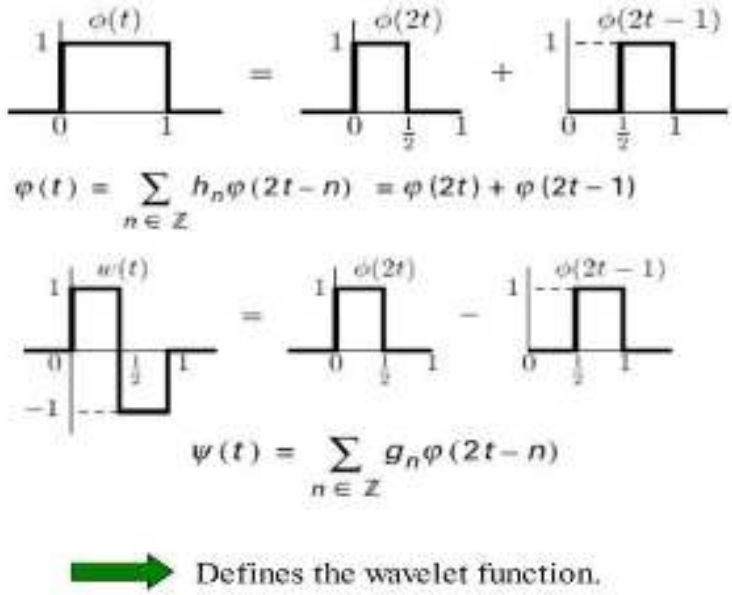








Two 2-scale relations:



Orthogonal wavelet bases (1)

- Find an orthogonal basis of V_j $\varphi j,k(t)=2^{\frac{j}{2}}\varphi\left(2^{j}t-k\right), k\epsilon z$
- Two-scale equation:

$$\varphi j,k=\frac{1}{\sqrt{2}}\sum_{n\epsilon z} hn\varphi j+1{,}2k+n\ ,\ \ \varphi(t)=\sum_{n\epsilon z} gn\varphi(2t-n)$$

- Orthogonal requires : $gn=(-1)^{n}h1-n$ $,\sum_{n} hn=2$

$$\sum_{n} hnhn+2k=2\ if\ k=0\ ,otherwisw=0$$

$$\sum_{n}(-1)^{n}n^{m}hn=0,m=0,\ldots\ldots,N-1$$

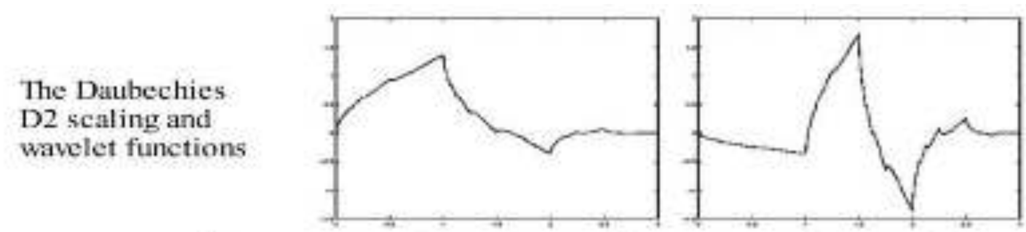
N: number of vanishing moments of the wavelet function

Orthogonal wavelet bases (2)

Other way around, find a set of coefficients t h_n atisfy the above equations.

Since the solution is not unique, other favorable properties can be asked for: compact support, regularity, number of vanishing moments of the wavelet function.

- Then solve the two-scale equations.
- Example: Daubechies seeks wavelets with minimum size compact support for any specified number of vanishing moments.



$$h_n = (\tfrac{1}{4}(1 + \sqrt{3}) \; \tfrac{1}{4}(3 + \sqrt{3}) \; \tfrac{1}{4}(3 - \sqrt{3}) \; \tfrac{1}{4}(1 - \sqrt{3}))$$

Orthogonal wavelet bases (2)

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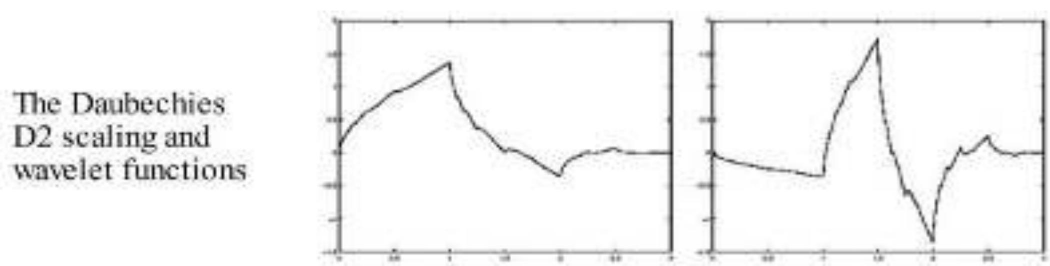


Most wavelets we use can't be expressed analytically.

Fast algorithms (1)

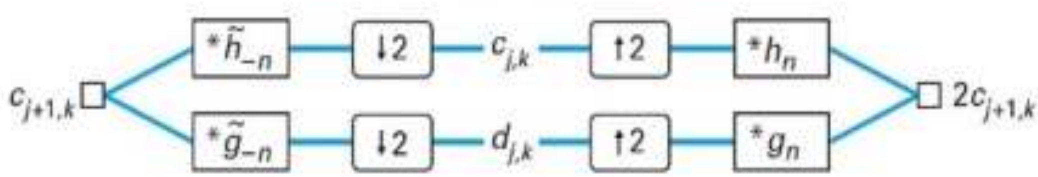
- We start with $f = \sum_{k-n}^{2^j-1} cj,k\varphi j,k$
- We want to obtain $f - co,o\varphi + \sum_{j=0}^{J-1} \sum_{k\epsilon z} dj,k\varphi j,k$
- We use the following relations between coefficients at different scales:

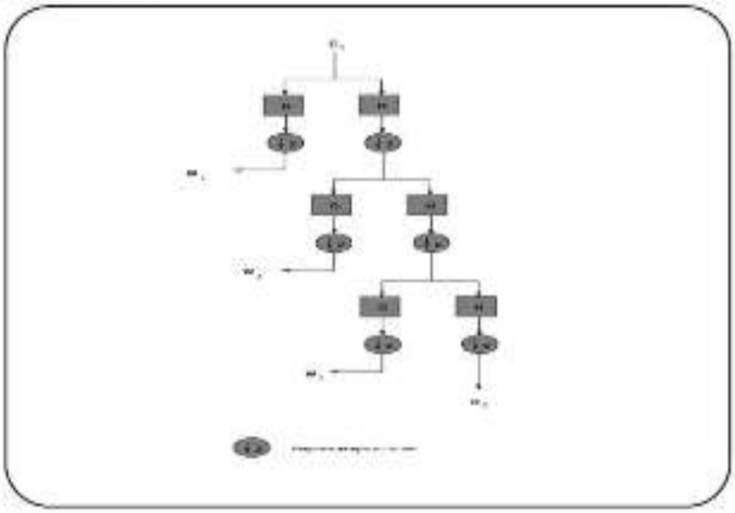
$$cj,k = \frac{1}{\sqrt{2}} \sum_{n \in z} nn - 2k^{cj} + 1,n \quad ,dj,k = \frac{1}{\sqrt{2}} \sum_{n \in z} gn = 1k^{cj} + 1,n$$

- Reconstruction is obtained with:

$$cj + 1,k = \frac{1}{\sqrt{2}} \sum_{n\epsilon z} cj,n^{hk-2n} + \frac{1}{\sqrt{2}} \sum_{n\epsilon z} dj,n^{gk-2n}$$

Fast algorithms using filter banks





2D Orthogonal wavelet transform

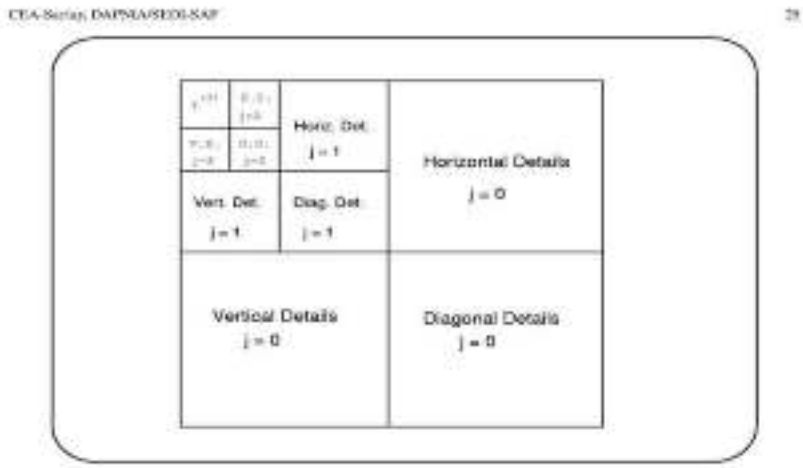
At two dimensions, we separate the variables x,y:

- vertical wavelet: $\psi^1(x,y) = \phi(x)\psi(y)$
- horizontal wavelet: $\psi^2(x,y) = \psi(x)\phi(y)$
- diagonal wavelet: $\psi^3(x,y) = \psi(x)\psi(y)$

The detail signal is contained in three sub-images

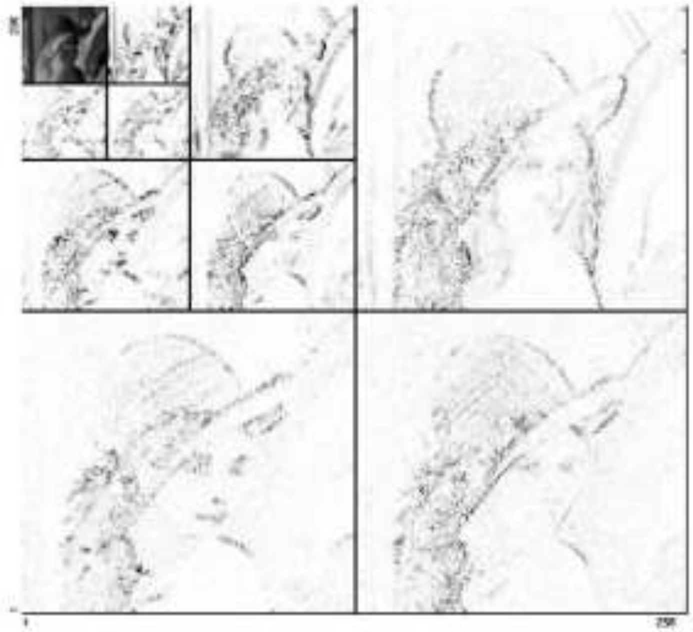
$$
\begin{aligned}
w_j^1(k_x,k_y) &= \sum_{l_x=-\infty}^{+\infty}\sum_{l_y=-\infty}^{+\infty} g(l_x-2k_x)h(l_y-2k_y)c_{j+1}(l_x,l_y)\\
w_j^2(k_x,k_y) &= \sum_{l_x=-\infty}^{+\infty}\sum_{l_y=-\infty}^{+\infty} h(l_x-2k_x)g(l_y-2k_y)c_{j+1}(l_x,l_y)\\
w_j^3(k_x,k_y) &= \sum_{l_x=-\infty}^{+\infty}\sum_{l_y=-\infty}^{+\infty} g(l_x-2k_x)g(l_y-2k_y)c_{j+1}(l_x,l_y)
\end{aligned}
$$

2D Orthogonal wavelet transform



Example :

Example :



Biorthogonal Wavelet Transform :

It is a generalization of the orthogonal wavelets. Two other spaces $\tilde{O}_j$ and $\tilde{V}_j$ are introduced for the reconstruction:

- $V_{j-1} = V_j \oplus O_j$, and $V_j \not\perp O_j$
- $\tilde{V}_{j-1} = \tilde{V}_j \oplus \tilde{O}_j$, and $\tilde{V}_j \not\perp \tilde{O}_j$
- $\tilde{O}_j \perp V_j$ and $O_j \perp \tilde{V}_j$

Biorthogonal Wavelet Transform:

Using two other filters ħ and ĝ, defined to be conjugate to h and g. The reconstruction of the signal is performed with:

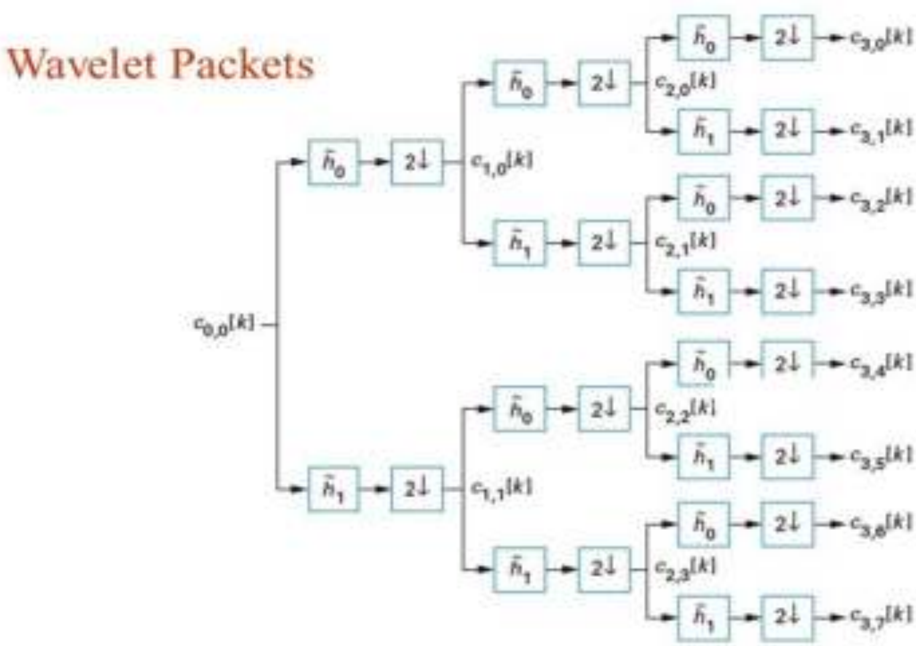
$$C_{j,k} = 2\sum_{l}[cj + 1, l\hbar(k - 21) + wj + 1, lg(k - 21)]$$

In order to get an exact reconstruction, two conditions are required for the conjugate filters:

- Dealiasing condition: $\hat{h}(v + 1/2)\hbar(v) + \hat{g}(v + 1/2)\hat{g}(v) = 0$

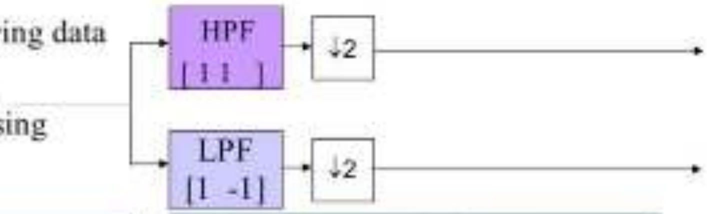
-Exact restoration: $h(v)\hbar(v) + \hat{g}(v)\breve{g}(v) = 1$

The structure of the filter bank algorithm is the same.



Example

Suppose you have the following data
$x(n) = [2 \quad 1 \quad 5 \quad 4]$
Find its wavelet transform using
Haar coefficient



HPF: using linear convolution

*	2	1	5	4
1	2	1	5	4
1	2	1	5	4

$x_h(n)=[\underline{2} \quad 3 \quad \underline{6} \quad 9 \quad \underline{4}]$

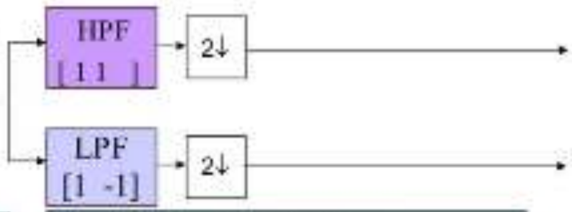
LPF: using linear convolution

*	2	1	5	4
1	2	1	5	4
1-	-2	-1	-5	-4

$x_l(n)=[\underline{2} \quad -1 \quad 4 \quad -1 \quad \underline{-4}]$

Example (1)

Suppose you have the following data
$x(n) = [2 \quad 1 \quad 5 \quad 4]$
Find its wavelet transform using
Haar coefficient



HPF: using linear convolution

*	2	1	5	4
1	2	1	5	4
1	2	1	5	4

$x_h(n)=[2 \quad 3 \quad 6 \quad 9 \quad 4]$

$x_h(n)=[3 \; 9 \quad]$

LPF: using linear convolution

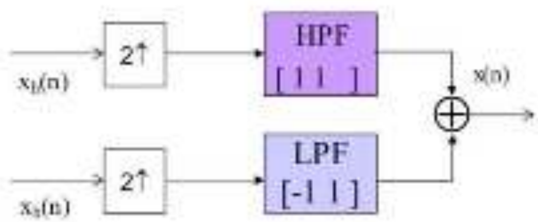
*	2	1	5	4
1	2	1	5	4
-1	-2	-1	-5	-4

$x_l(n)=[2 \quad -1 \quad 4 \quad -1 \quad -4]$

$x_l(n)=[-1 \quad -1]$

Example (2)

Decompose the following signals:
$x_h(n)={}^1/_{\sqrt{2}}[3 \quad 9]$
$x_l(n)={}^1/_{\sqrt{2}}[-1 \quad -1]$



HPF: using linear convolution
$x_h(n)={}^1/_{\sqrt{2}}[3 \quad 0 \quad 9 \quad 0]$

*	3	0	9	0
1	3	0	9	0
1	3	0	9	0

$={}^1/_2\,[3 \quad 3 \quad 9 \quad 9]$

LPF: using linear convolution
$x_l(n)={}^1/_{\sqrt{2}}[-1 \quad 0 \quad -1 \quad 0]$

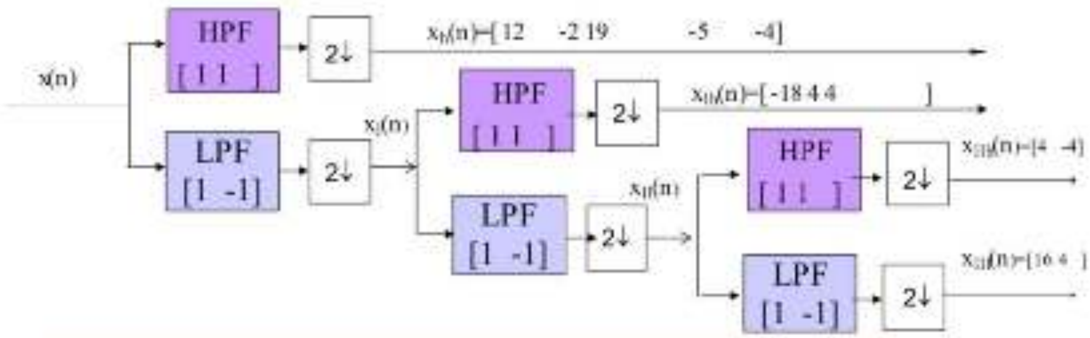
*	-1	0	-1	0
-1	1	0	1	0
1	-1	0	-1	0

$={}^1/_2\,[1 \quad -1 \quad 1 \quad -1]$

Summation $x(n) = {}^1/_2\,[4 \quad 2 \quad 10 \quad 8]$

$= [2 \quad 1 \quad 5 \quad 4]$

Multilayer Wavelet Transform



HPF: using linear convolution

*	9	3	5	-7	11	8	-5	2	-4
1	9	3	5	-7	11	8	-5	2	-4
1	9	3	5	-7	11	8	-5	2	-4

$x_h(n)$=[9 12 8 -2 4 19 3 -3 -2 -4]
$x_h(n)$=[12 -2 19 -3 -4]

Multilayer Wavelet Transform

LPF: using linear convolution

*	9	3	5	-7	11	8	-5	2	-4
1	9	3	5	-7	11	8	-5	2	-4
-1	-9	-3	-5	7	-11	-8	5	-2	4

$x_l(n)$=[9 -6 2 -12 18 -3 -13 7 -6 4]
$x_l(n)$=[-6 -12 -3 7 4]

HPF: using linear convolution

*	-6	-12	-3	7	4
1	-6	-12	-3	7	4
1	-6	-12	-3	7	4

$x_{llh}(n)$=[-6 -18 -15 4 11 4]
$x_{llh}(n)$=[-18 4 4]

LPF: using linear convolution

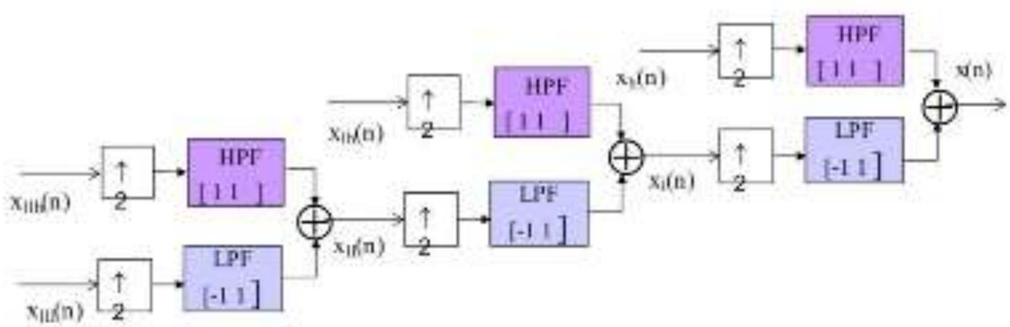
*	-6	-12	-3	7	4
1	-6	-12	-3	7	4
-1	6	12	3	-7	-4

$x_{ll}(n)$=[-6 -6 9 10 -3 -4]
$x_{ll}(n)$=[-6 10 -4]

HPF:
$x_{lllh}(n)$=[-6 4 6 -4]=[4 -4]

LPF::
$x_{lll}(n)$=[-6 16 -14 4]=[16 4]

Multilayer Inverse Wavelet Transform



HPF:
$x_{ll}(n)={}^1/_{\sqrt{2}}$ [4 0 -4 0]

*	4	0	-4	0
1	4	0	-4	0
1	4	0	-4	0

$= {}^1/_2\,[4 \quad 4 \quad -4 \quad -4]$

LPF:
$x_{ll}(n)={}^1/_{\sqrt{2}}$ [16 0 4 0]

*	16	0	4	0
-1	-16	0	-4	0
1	16	0	4	0

$= {}^1/_2\,[-16 \quad 16 \quad -4 \quad 4]$

Summation: $x_{ll}(n) = {}^1/_2\,[-12 \quad 20 \quad -8 \quad 0] = [-6 \quad 10 \quad -4]$

Multilayer Inverse Wavelet Transform

HPF:

$x_H(n) = {}^1/_{\sqrt{2}}$ [-18 0 4 0 4 0]

*	-18	0	4	0	4
1	-18	0	4	0	4
1	-18	0	4	0	4

$= {}^1/_2 [-18 \quad -18 \quad 4 \quad 4 \quad 4]$

LPF:

$x_L(n) = {}^1/_{\sqrt{2}}$ [-6 0 10 0 -4 0]

*	-6	0	10	0	-4
-1	6	0	-10	0	4
1	-6	0	10	0	-4

$= {}^1/_2 [6 \quad -6 \quad -10 \quad 10 \quad 4]$

Summation: $x_l(n) = {}^1/_2 [-12 \quad -24 \quad -6 \quad 14 \quad 8] = [-6 \quad -12 \quad -3 \quad 7 \quad 4]$

HPF:

$x_0(n) = {}^1/_{\sqrt{2}}$ [-18 0 4 0 4 0]

*	12	0	-2	0	19	0	-3	0	-4
1	12	0	-2	0	19	0	-3	0	-4
1	12	0	-2	0	19	0	-3	0	-4

$= {}^1/_2 [12 \quad 12 \quad -2 \quad -2 \quad 19 \quad 19 \quad -3 \quad -3 \quad -4 \quad -4]$

LPF:

$x_l(n) = {}^1/_{\sqrt{2}} [-6 \quad 0 \quad -12 \quad 0 \quad -3 \quad 0 \quad 7 \quad 0 \quad 4]$

*	-6	0	-12	0	-3	0	7	0	4
-1	6	0	12	0	3	0	-7	0	-4
1	-6	0	-12	0	-3	0	7	0	4

$= {}^1/_2 [6 \quad -6 \quad 12 \quad -12 \quad 3 \quad -3 \quad -7 \quad 7 \quad -4 \quad 4]$

Summation: $x(n) = {}^1/_2$ [18 6 10 -14 22 16 -10 4 -8]=[9 3 5 -7 11 8 -5 2 -4]